SIMILITUDES & DIFFERENCES

AMONG ADM

BSSN

BONA - MASSÓ

EINSTEIN - CHRISTOFFEL

EVOLUTION SYSTEMS

Carles Bona
Palma University

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	Metric variables	X-tra variables
2nd ORDER	ADM	BSSN
1st ORDER	EINSTEIN-CHRISTOFFEL	BONA-MASSÓ

EVOLUTION SYSTEM

$\downarrow$

$\boxed{\text{CONSTRAINT EQ.}}$

$\downarrow$

MODIFIED EVOLUTION SYSTEM

ADM SYSTEM

3+ $\quad d^{2} = \alpha^{2} dt^{2} + \gamma_{i} (dx^{i} + \beta dt)(dx + \beta dt)$

CONSTRAINTS

$^{3)}R \quad tr(K^{\alpha} \quad + (tr = \quad ^{2} \quad 2\,Z \qquad \mathcal{C}$

$\nabla_k K^k \quad \partial_k (tr K \quad S \qquad \mathcal{C}$

EVOLUTION

$(\partial_t - \mathcal{L}_\beta)\gamma_{ij} = 2 \quad K_i$

$(\partial_t - \mathcal{L}_\beta) K_{ij} = \nabla_i \alpha_j \quad \alpha [\, ^{3)}R_i \quad 2K^2 \quad tr K K_i \quad \hat{R}_i \,]$

$n \frac{\alpha}{4} \mathcal{C} \gamma_{ij}$

COVARIANT FRAMEWORK

DEFINE A ZERO VECTOR Z

$$(\partial_t \quad \mathcal{L}_\beta \quad Z \qquad \alpha\, [\, \nabla_k (K^k \quad \delta^k \, \mathrm{tr} K \qquad S_k \,]$$

→ system deviation

MODIFY IN A COVARIANT WAY

$$\boxed{(\partial_t - \mathcal{L}_\beta) K_{ij} =} \; \nabla_i d_j \alpha + \alpha\, [\, {}^{(3)}R_{ij} \quad K^2 + \mathrm{tr}K \, K_{ij} \qquad \hat{R} \quad]$$

$$\frac{\mu}{4}\, \alpha\, [\, {}^{(3)}R \quad \mathrm{tr}K \quad + \mathrm{tr}^2 K) \quad ZZ \,]\, \gamma_{ij}$$

$$+ \frac{\mu}{2}\, \alpha \quad \nabla_i Z \quad \nabla_j Z \qquad \frac{\nu}{2}\, \alpha\, (\nabla Z \quad \gamma_{ij}$$

ν free parameters

COVARIANT UNDER $\quad t \quad f(t)$

$(x \quad ' \quad h(x \quad t)$

Use it to take $\beta^i \quad 0$)

COMPARE WITH BSSN

CONFORMAL DECOMP

$$A_{ij} \quad \gamma^{1/3}(K_{ij} \quad \tfrac{1}{3}\mathrm{tr}K\,\gamma_{ij}$$

$$\tilde{\Gamma} \quad 2\gamma^{1/3}[\,! \, D^{k}{}_{k} \quad D_{k}{}^{ik}$$

$$D_{kij} \quad {}_{2}\,\partial_k \gamma_{ij}$$

SET OF VARIABLES $\quad \hat{A}_{ij} \quad \mathrm{tr}K, \hat{\Gamma}$

not a vector

vs $\quad K_{ij} \quad Z$

QUASIEQUIVALENCE CONDITIONS

- $\mathrm{tr}(\hat{A}_{ij}) \equiv 0$ — not a variable
- $\tilde{\Gamma}^{i} = -2\gamma^{1/3}[\tfrac{1}{3}D^{ik}{}_{k} - D_{k}{}^{ik} - Z$
- $\mu = 2, \quad \nu = n = 4/3$

QUASIEQUIVALENT $\quad$ PRINCIPAL PARTS EQUIVALENT

EQUIVALENT $\quad \equiv \quad$ IDENT CA UP TO A CHANGE OF VARIABLES

COMPARING WITH BONA-MASSO

FIRST ORDER VERSION $\quad K_{ij} \quad D_{kij} \quad A_k \quad V_k$

- $D_{kij} = \frac{1}{2}\partial_k \gamma_{ij} \qquad A_k \quad \partial_k \ln \alpha$

$$\partial_t D_{kij} + \partial_k (\alpha \gamma_{ij} \quad \circ$$
$$\partial_t A_k + \partial_k (\alpha Q \quad \circ$$

$(Q \qquad \partial_t \ln \alpha$

- ${}^{(3)}R_{ij}$ DECOMPOSITION $\qquad$ ordering ambiguity

$\partial D_k{}_s \ \text{vs} \ \partial_k D_j{}_s$

$V^i \quad D^k{}_k \quad D^{k}{}_k \qquad$ not a vector

EQUAS EQUIVALENCE CONDITIONS

SE THE SAME RICCI DECOMPOSITION

$V^i \quad D^k{}_k \quad D^k{}_k \quad Z^i$

$$\boxed{\mu = 2, \ \nu = n}$$

COMPARING WITH EINSTEIN-CHRISTOFFEL

NO XTRA QUANTITIES K_{ij} D_{kij} A_k

densitized lapse

MODIFIED EVOLUTION FOR D_{kij}

$\partial_t D_{kij} + \partial_k (\alpha K_{ij}$ $\alpha (C_i \gamma_{jk} + C_j \gamma_{ik})$

QUASIEQUIVALENCE CONDITION

$\partial_t \tilde{D}_{kij} + \partial_k (\alpha K_{ij})$ 0

$\tilde{D}_{kij} = D_{kij} + \gamma_{ki} Z_j + \gamma_{kj} Z_i$

- USE $\tilde{D}_{kij}$ TO REPLACE D_{kij} EVERYWHERE
- $\mu = 2$ ν 0