NONLINEAR HARMONIC EVOLUTION

$$\gamma^{\alpha\beta} = \sqrt{-g}\, g^{\alpha\beta} \qquad \Gamma^{\alpha} = -\Box x^{\alpha}$$

$$G^{\mu\nu} = \mathcal{E}^{\mu\nu} + B^{\mu\nu} \qquad \Gamma^{\alpha} = 0 \Rightarrow B^{\mu\nu} =$$

REDUCED EQUATIONS

$$\mathcal{E}^{\mu\nu} = \underbrace{-\frac{1}{2g}\gamma^{\alpha\beta}\partial_\alpha\partial_\beta\gamma^{\mu\nu}}_{\text{principal part}} + S^{\mu\nu} =$$

FISHER-MARSDEN

$$u = (\gamma^{\alpha\beta}, T^{\alpha\beta}, X^{\alpha\beta}, Y^{\alpha\beta}, Z^{\alpha\beta})$$

$$\mathcal{E}^{\mu\nu} = 0 \rightarrow A^{\alpha}\partial_\alpha u = S(u)$$ <u>Well Posed</u>

HARMONIC CONSTRAINTS

$$v = (\Gamma^{\alpha}, \partial_\beta \Gamma^{\alpha})$$

Bianchi identities $\Rightarrow \tilde{A}^{\alpha}\partial_\alpha v = \tilde{S}(u)\, v$

Uniqueness $\Rightarrow$

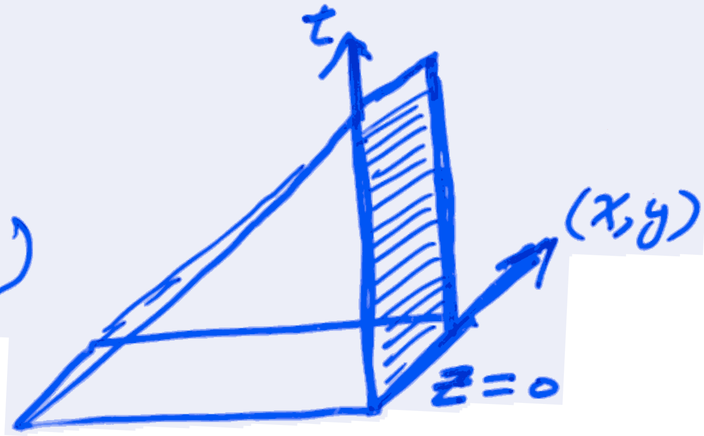
Initial Data
$v=0$

WELL-POSED INITIAL-BOUNDARY PROBLEM

HOMOGENEOUS BOUNDARY DATA

ADAPT COORDINATES TO BOUNDARY

$x^a = (t, x, y, z) = (x^a; z)$



SET $\gamma^{za} = 0 \Rightarrow F^z = -\sum T^{\alpha\beta} Z^{\alpha\beta}$

Maximally Dissipative Condition [1]

$$Mu = 0 \Rightarrow F^z \geq 0$$

M independent of u

REDUCED EQUATIONS WELL-POSED FOR DIRICHLET, NEUMANN, ...

REQUIRE CHOICE SO THAT THE HARMONIC CONDITIONS $\Gamma^\alpha = 0$

1. See Friedrich-Nagy for first use in GR

WELL-POSED INITIAL-BOUNDARY EVOLUTION

HOMOGENEOUS BOUNDARY DATA

Dirichlet $\quad \gamma^{za} = 0$

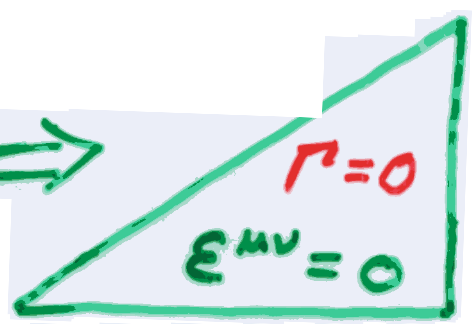
Neumann $\quad Z^{ab} = Z^{zz} = 0$

$$\left.\begin{array}{l} \gamma^{za} = 0 \\ Z^{ab} = Z^{zz} = 0 \end{array}\right\} \; Mu = 0$$

$$\text{at boundary} \Longrightarrow \left\{\begin{array}{l} \Gamma^{z} = 0 \\ \partial_z \Gamma^{a} = \partial_z^2 \gamma^{az} \end{array}\right.$$

REMARKABLY $\quad \mathcal{E}^{za} = 0 \Longrightarrow \partial_z^2 \gamma^{az} = 0$

UNIQUENESS $\Longrightarrow$



$Mu = 0$

CONSTRAINTS $= 0$

$\Gamma^{\alpha} = 0$

WELL-POSEDNESS OF EINSTEIN'S EQUATIONS

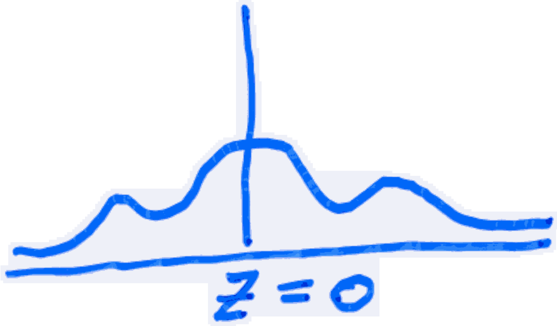
BOUNDARY DATA

HOMOGENEOUS

$M u = 0$

EVEN PAR TY

UNPHYS CAL



INHOMOGENEOUS

$Mu = g(x^a)$

free boundary data

ROBUST STABILITY

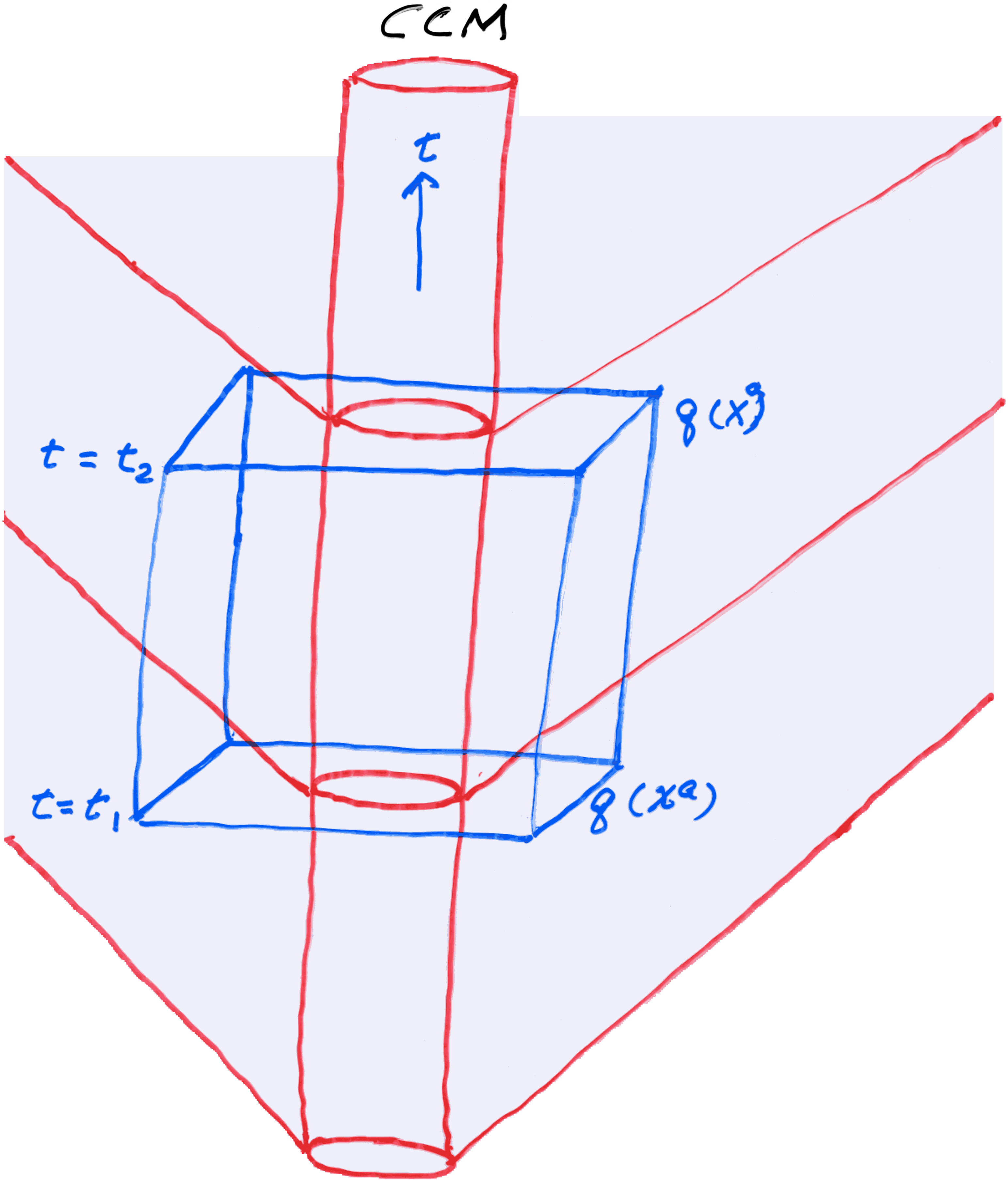
$g(x^a)$ random

IMPLEMENTED AS ROBUSTY STABLE CONVERGENT NONLINEAR CODE

PHYS CAL DATA

$g(x^a)$ from CC

ROBUST IMPLEMENTATION OF CC USING L NEAR ZED CODES



$g(x^a)$ interpolated from characteristic grid