NR @ Penn State

Black Hole Head-On Collisions Revisited

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Short History of Binary BH Mergers

- Binary Mergers simulations have been attempted for years:
- Axisymmetry:
  - Smarr, Eppley (late 1970s), Shapiro and Teukolsky (early 1990s), Anninos et al.(early 1990s), Choptuik et al. (2003)
  - done with conformally flat, Brill-Lindquist and Bowen-York-type data
- Full 3D:
  - AEI (1999): puncture data, vanilla ADM evolution system
  - AEI (2001): puncture data, BSSN evolution system
- Full 3D, with Singularity Excision
  - PSU, U. Pitt, U.T. Austin (2000)
  - Present study is an extension of latter work

Motivation – Why Do This Again?

- BBH Problem is a physically relevant scenario
- Can test dynamic excision in Maya code
- Can test feedback techniques for motion
- Expand on previous Kerr-Schild-based work

The Maya Code

- Successor to "Agave" (post-Grand Challenge code)
  - Originally written by E. Schnetter
  - Added to by Fiske, Kelly, Laguna, Shoemaker, Smith, Sperhake
- Very basic NR evolution code:
  - 3D, Cartesian coordinates
  - Uniform resolution
- Disadvantages:
  - Coordinates not adapted to any problem
  - Boundaries uncomfortably close
  - Can't deal with different length scales
- Advantages:
  - Coordinates not adapted to any problem
  - Boundaries uncomfortably close

Evolution Scheme

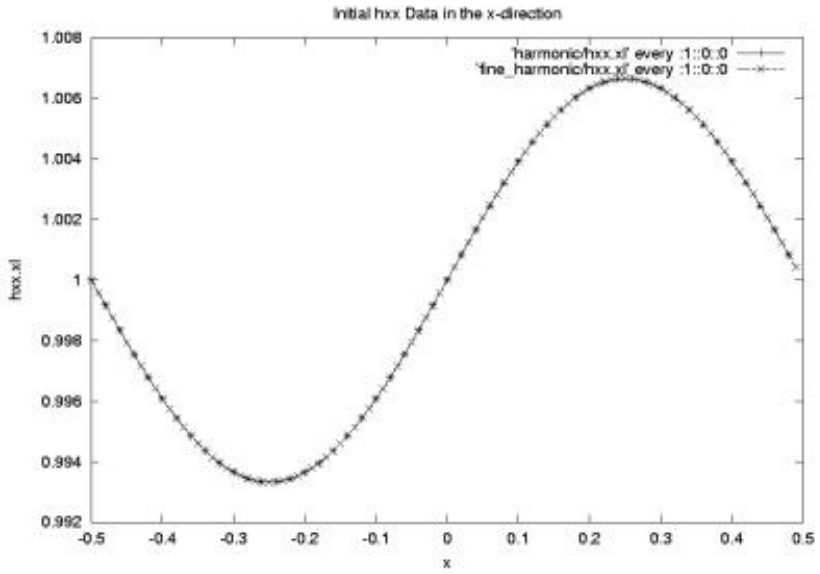
- **ICN evolution, using BSSN system of equations:**
  - **Shibata and Nakamura (1995), Baumgarte and Shapiro (1999)**
  - **Optional Yo et al. (2002) terms for stability**

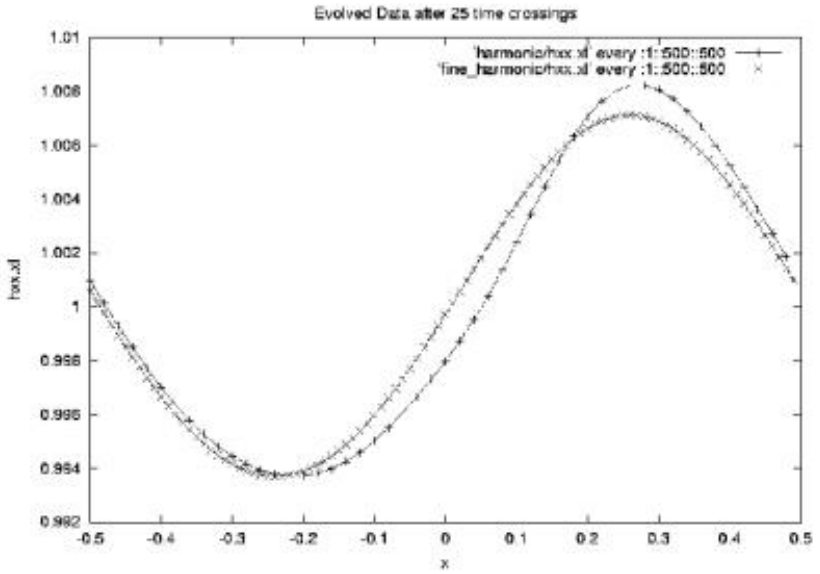
$$
\begin{aligned}
(\partial_t - \mathcal{L}_\beta)\,\tilde{\gamma}_{ij} &= -2\alpha\tilde{A}_{ij}, \\
(\partial_t - \mathcal{L}_\beta)\,\phi &= -\frac{1}{6}\alpha K, \\
(\partial_t - \mathcal{L}_\beta)\,\tilde{A}_{ij} &= e^{-4\phi}[-D_iD_j\alpha + \alpha R_{ij}]^{TF} \\
&\quad + \alpha(K\tilde{A}_{ij} - 2\tilde{A}_{ik}\tilde{A}^k{}_j), \\
(\partial_t - \mathcal{L}_\beta)\,K &= -D^iD_i\alpha \\
&\quad + \alpha(\tilde{A}_{ij}\tilde{A}^{ij} + \frac{1}{3}K^2), \\
\partial_t\tilde{\Gamma}^i &= \tilde{\gamma}^{jk}\partial_j\partial_k\beta^i + \frac{1}{3}\tilde{\gamma}^{ij}\partial_j\partial_k\beta^k + \beta^j\partial_j\tilde{\Gamma}^i - \tilde{\Gamma}^j\partial_j\beta^i \\
&\quad + \frac{2}{3}\tilde{\Gamma}^i\partial_j\beta^j - \left(\chi + \frac{2}{3}\right)\left(\tilde{\Gamma}^i - \tilde{\gamma}^{jk}\tilde{\Gamma}^i_{jk}\right)\partial_j\beta^j \\
&\quad - 2\tilde{A}^{ij}\partial_j\alpha + 2\alpha(\tilde{\Gamma}^i{}_{jk}\tilde{A}^{jk} + 6\tilde{A}^{ij}\partial_j\phi - \frac{2}{3}\tilde{\gamma}^{ij}\partial_j K)
\end{aligned}
$$

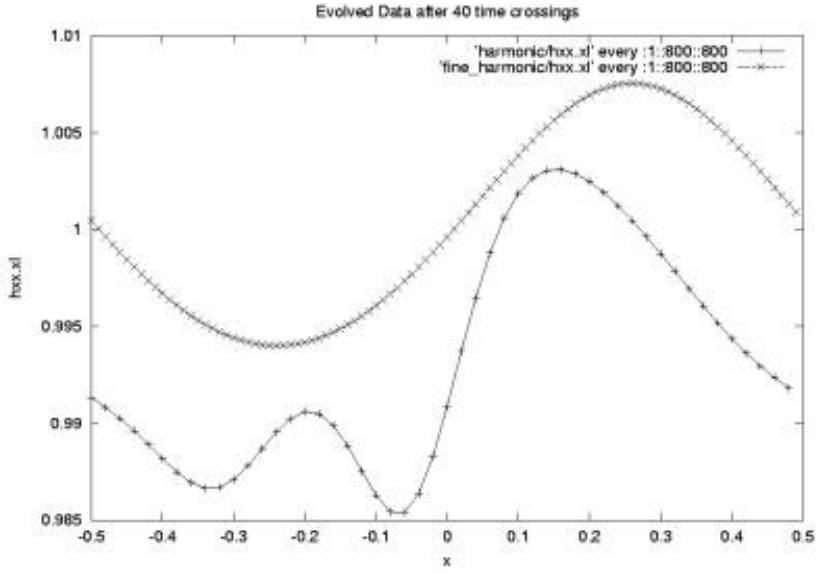
Apples with Apples for Maya (T Landis, BK)

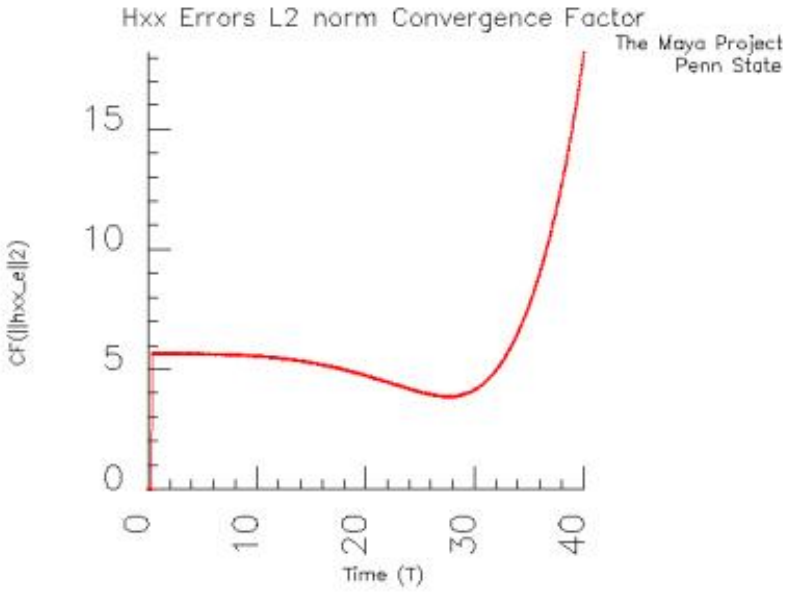
- **Carried out GaugeWave tests in Maya for vanilla BSSN system**
- **gauge four-metric:** $ds^2 = -H\,dt^2 + H\,dx^2 + dy^2 + dz^2$
  - **sinusoidal:** $H(u) = 1 + A \sin\left(\frac{2\pi u}{d}\right)$
  - **gaussian:** $H(u) = 1 + A \exp\left(-\frac{u^2}{d^2}\right)$

AwA results – Sinusoidal h_{xx} ($0T$, $25T$, $40T$, converge)

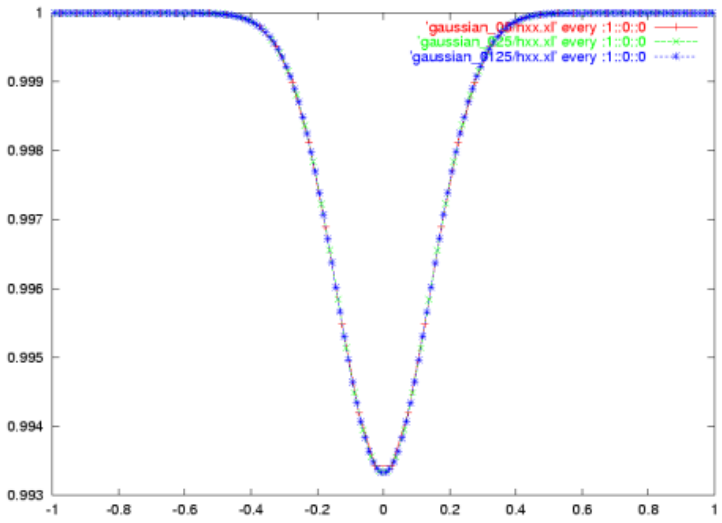


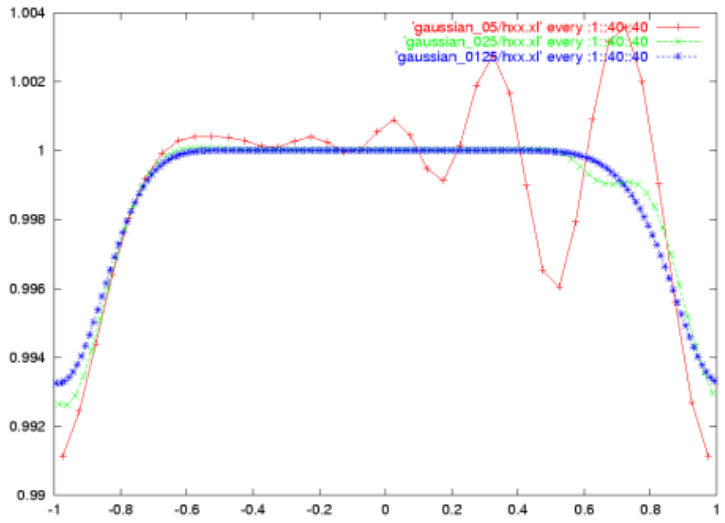


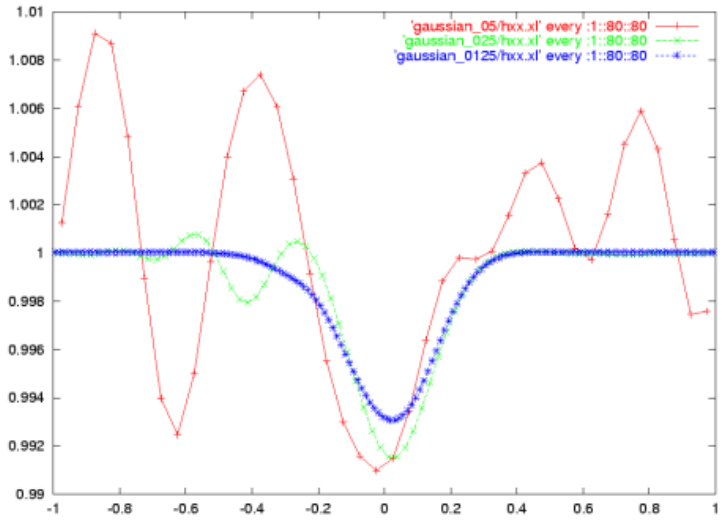


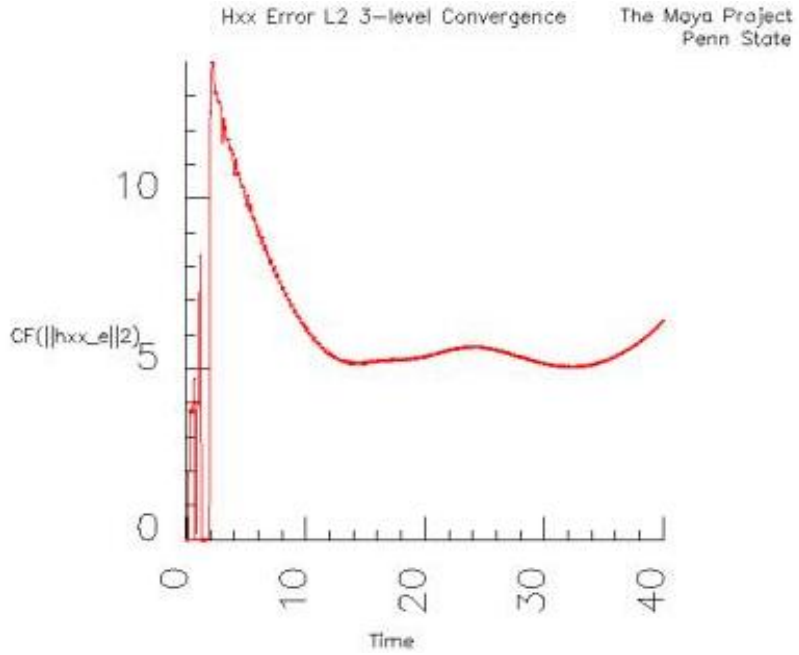


AwA results – Gaussian h_{xx} ($0T$, $2.5T$, $5T$, converge)









Binary Initial Data

- **Blended Kerr-Schild-type metric:**
  - **Matzner, Huq, Shoemaker (1998)**

$$
\begin{aligned}
\gamma_{ij} &= {}^{A}\gamma_{ij} + {}^{B}\gamma_{ij} - \eta_{ij} \\
K_{ij} &= \gamma_{(i|m} \left({}^{A}K^{m}_{\ |j)} + {}^{B}K^{m}_{\ |j)} \right) \\
\alpha &= {}^{A}\alpha \, {}^{B}\alpha \\
\beta^{i} &= {}^{A}\beta^{i} + {}^{B}\beta^{i}
\end{aligned}
$$

- **Initially boosted, non-spinning black holes**
- **Constraints solved in Conformal Transverse-Traceless decomposition**

Warm-Up: On-Top Evolution

- **Binary data within single common AH**
  - **Reason:**
  - **tests robustness of evolution system, gauge conditions**
  - **avoids issues of moving excision, tracking**
- **Treat evolution as for "circling" black holes [gr-qc/0307015]**
  - **BSSN system with Yo trick**
  - **live slicing conditions (1+log, gamma driver shift)**
  - **densitized lapse**
- **Use spherical excision**
- **Use outgoing boundary conditions for everything except $\tilde{\Gamma}^i$:**

$$u = u_0 + \frac{w(t-r)}{r^n}$$

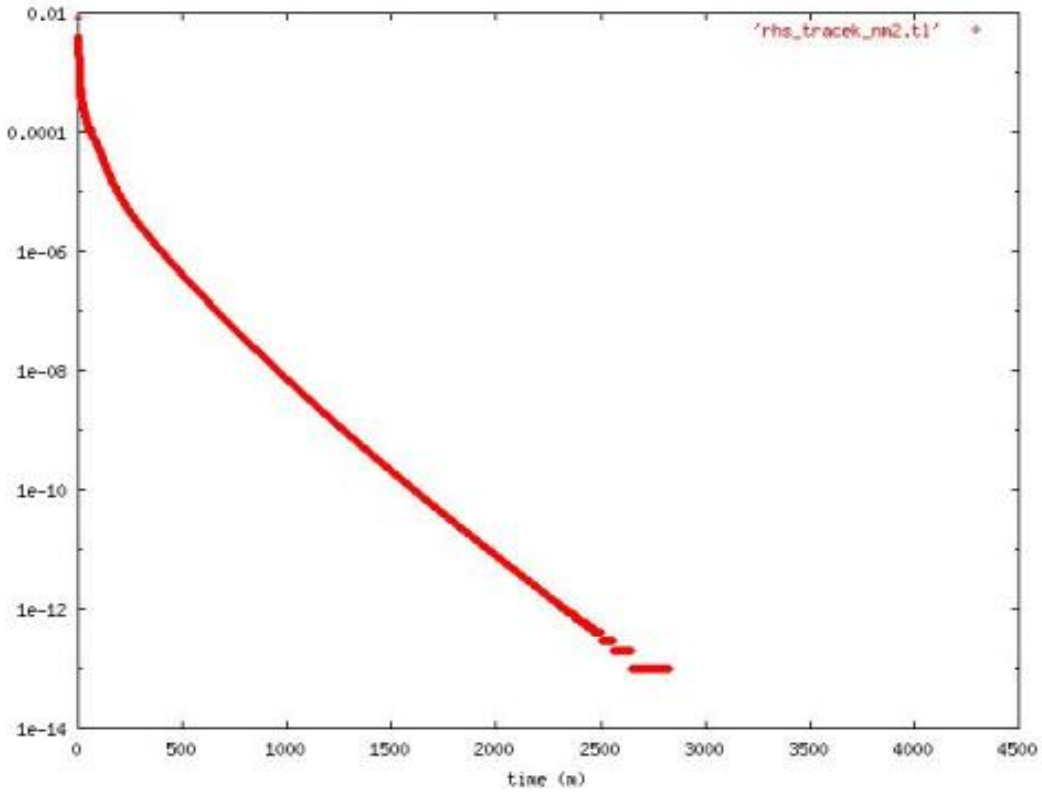
$$\Rightarrow \partial_t u = -\frac{x^i}{r}\partial_i(u-u_0) - \frac{n(u-u_0)}{r}$$

Other messy facts

- **coord separation of** $3.0m$
- $\Delta = 0.25m$**, octant symmetry, size** $\geq (12m)^3$
- **outer b.c. is relative to IEF solution – need IEF mass**
  - **judge from AH area**
  - **or** ψ_2 **falloff at late times**
  - **or ADM mass at late times**
  - **or other iterative scheme**

On-Top Results

L2-norm of $\partial_t K$



Movie - $\hat{C}_{Ham}$ **(normalized Hamiltonian constraint)**

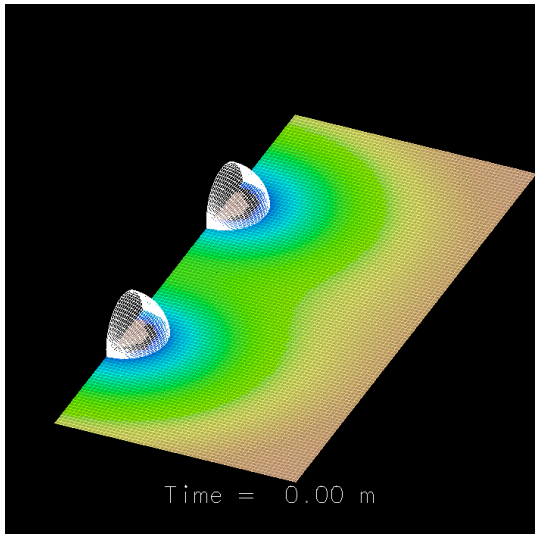
Main Event: Head-On Collision

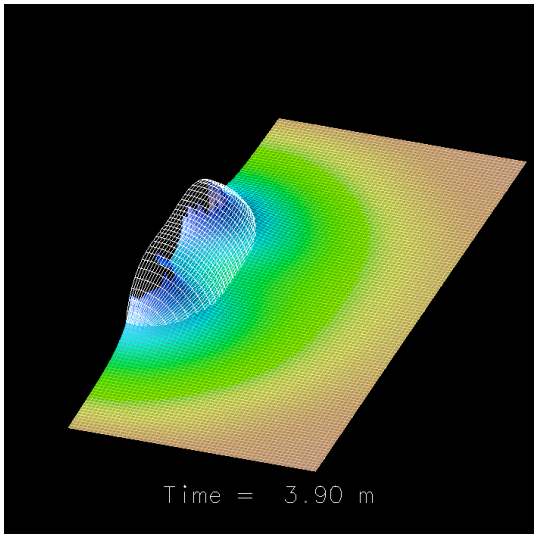
- Coordinate center-to-center separation of $10m$
- Use pseudo-Newtonian trajectory – boost, constant acc
- Dynamic excision of singularities
- Tracking of field gradients near excision boundaries
  - gives good centering for updated excision mask
- Merge excision masks when close enough
- Switching of gauge conditions after merger
  - before merger, analytic gauge conditions, outgoing outer boundaries
  - after merger, switch to On-Top-like conditions

Head-On - Apparent Horizons and Merger

K – trace of extrinsic curvature

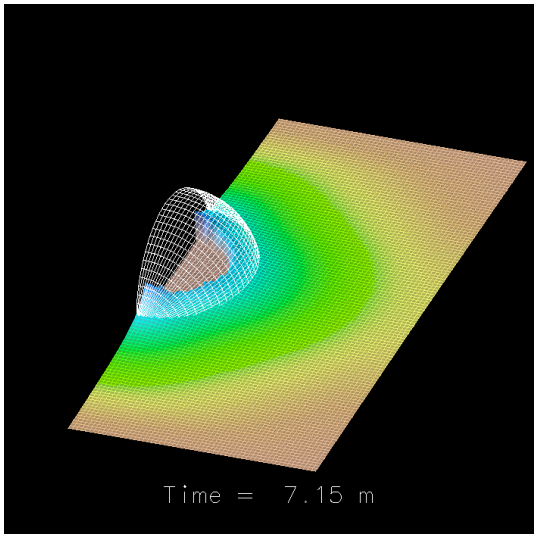
initial data





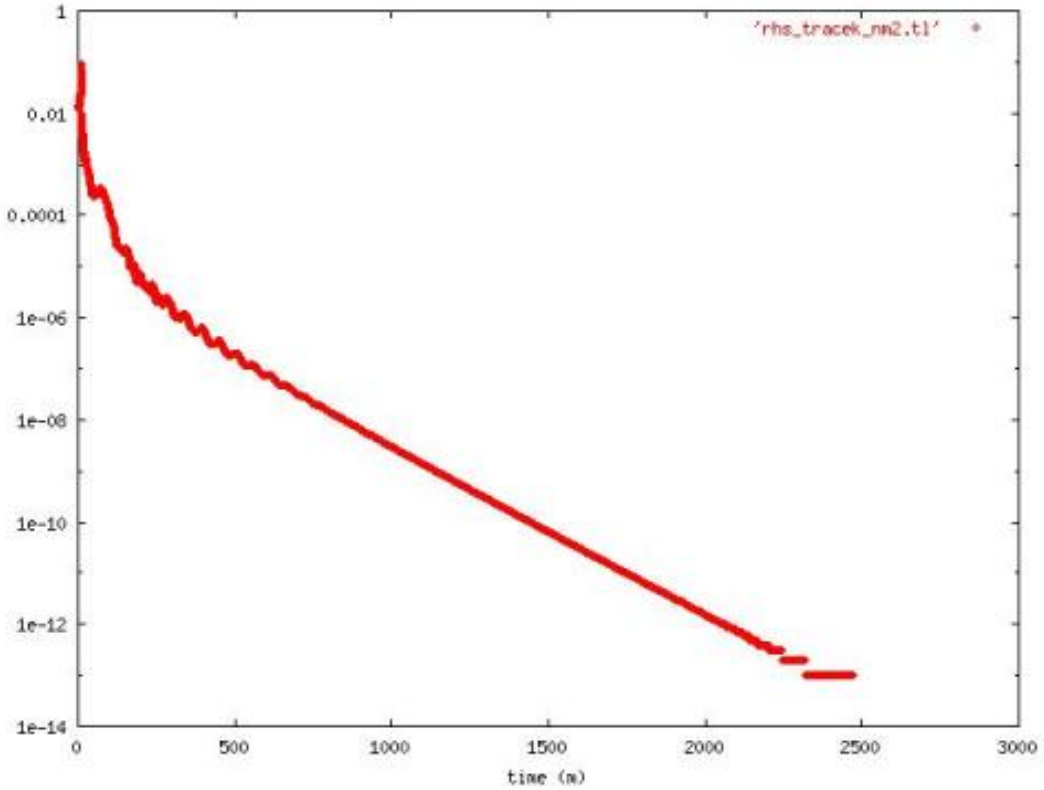
common AH

post merge



Head-On Results

L2-norm of $\partial_t K$



Movie - K

Movie - $\hat{C}_{Ham}$

Conclusions I

- **Earlier K-S excised runs lasted** $\sim 40m$**. Why so much better now?**
- **BSSN instead of ADM:** $\rightarrow\sim 90m$
- **densitized lapse:** $\rightarrow\sim 400m$
- **correctly symmetrized** K_{ij}**:** $\rightarrow\ 3000m$

Conclusions II

- **Achieved So Far**
  - **Successfully evolved boosted binary system head-on**
  - **System has common Apparent Horizon after** $\sim 4m$
  - **Runs go for** $3000m$ **or better – errors damp to machine precision**
- **What's Still Wrong**
  - **Using too much foreknowledge of singularities**
  - **Constraint violations from outer boundaries**
  - **Domain very small (**$\sim (18m)^3$**)**
  - **Can't handle very different masses**
- **In Progress**
  - **Larger domains**
  - **Larger initial separations (**$d = 16.0m$ **works)**
  - **Waveform extraction**
  - **Better outer boundary conditions**
  - **More efficient feedback mechanism for singularity positions**
  - **Better gauge conditions – less need for feedback**

Conclusions III

- **The Future?**
  - **Grazing and other non-head-on mergers**
  - **15 - 20 complete orbits of BBH system, leading to plunge, merger, and ringdown**
  - **World domination**