Test-suites for numerical relativity

Denis Pollney

Max-Plank-Institut für Gravitationsphysik
(Albert-Einstein-Institut)
Golm, Germany

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Abstract

The first Mexico meeting on testing evolution systems for Einstein's equations resulted in a suite of tests, using periodic boundary conditions, for which a large class of evolution systems could be compared. This talk motivates the need for standardised tests; outlines issues that arose in developing the `mexico1` test-suite; describes the tests that were finally chosen for `mexico1`; and opens the discussion for the design of a suite of more advanced tests for numerical relativity codes.

Why do we need test-suites for numerical relativity?

We currently enjoy a wealth of proposals for how to fix numerical relativity – and suffer from an equal number of claims which need to be verified.

Some successes with BSSN, a modified ADM-type formulation of Einstein's equations, has spurred development of a number of alternative suggestions for curing instabilities through better formulations.

A paper can only present a specific set of tests – but by selecting tests, it can also ignore important aspects of a formulation.

It is impossible for one group to check into all of these things themselves – need to act on information that they can trust.

Implementing a new set of equations numerically is a big effort – a lot of this time is tied up in testing, duplication of effort.

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A drug is not allow onto the market until it has passed some basic tests. (But even a well tested drug might produce unexpected side-effects.)

Folklore vs. established knowledge

Much of the knowledge in numerical relativity is tied up in “folklore”

- papers present the successes, say little about the failures
- some techniques are quite important, but only briefly mentioned and easily overlooked, or not mentioned at all – “tricks”
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Making this knowledge-though-experience more explicit will help bring scientists into the field more quickly.

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(1) Tests should be reasonably generic.

A number of factors stand in the way.

- different domains of integration, eg.:
  - ⋆ choice of coordinates (eg. polar vs. cartesian)
  - ⋆ built-in topology (eg. interior excision boundaries)
- different numerical techniques:
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(2) Tests need to define some criteria for “correctness”.

Exact solutions are ideal – if only we knew some useful ones.

Constraints are obvious tests – but not the only ones.

Physical scales can be important where there is no mathematical background.

- a plane wave should preserve it’s phase and amplitude
- radiated energy should be accounted for
- event horizons shouldn’t decrease in size

The output should be unambiguous.

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(3) Tests should try to decouple different aspects of the problem.

Eg. The interaction of boundary conditions with the interior can induce instabilities.

- *can we study the interior formulation independent of the boundaries?*
- *can we find ways of detecting bc-induced instabilities?*

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(4) Tests should be accessible.

A good test suite will run with minimal extra bells and whistles (excision, horizon finders, wave extraction, etc.).

- these things are difficult to implement and won't be available to everyone
- slight differences in implementation might cloud the picture

However, more complicated tests (black holes) are likely to require at least some extra infrastructure for analysis.

What makes a good test suite?

(5) Tests should try to model important problems.

At the end of the day, we are interested in binary black holes.

We should look for tests which are not necessarily physical, but which model important aspects of black holes:

- compact sources, asymptotically flat spacetimes
- large gradients in certain variables
- horizons
- etc.

Choices made for the `mexico1` tests

The `mexico1` tests were chosen to isolate features of the interior evolution formulation, independent of the interaction with outer boundaries.

(1) Test cases were limited to periodic solutions.

- eliminates interesting strong-field test cases such as brill waves and black holes
- some codes may be particularly disadvantaged by periodic boundaries (eg. the suggestion that BSSN propagates error modes off the grid)

(2) Where possible, the tests use harmonic slicing.

- an easy to specify/implement gauge condition, which should be available to most codes, nice analytic properties

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The `mexico1` testsuite: Four tests made the final cut:

- Robust stability
- Gauge wave
- Linear wave
- Gowdy spacetime

Robust stability

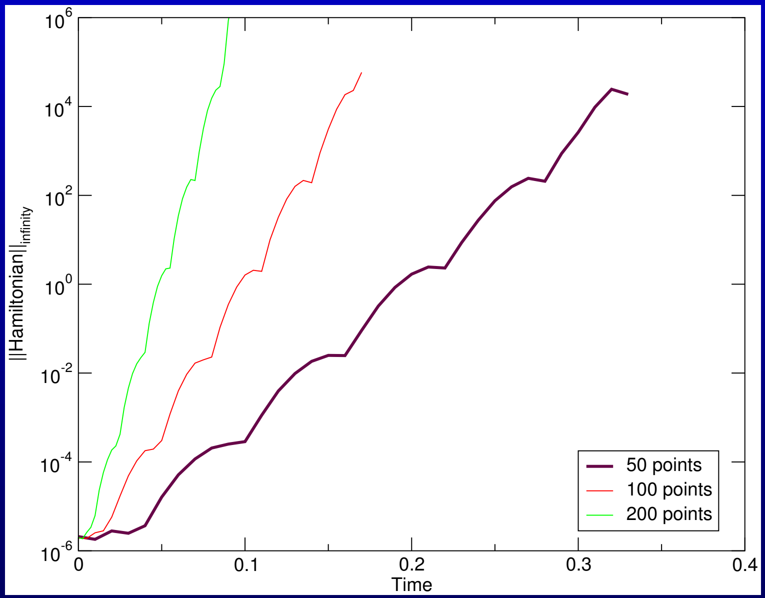
Random noise due to numerical error is unavoidable in any initial data calculation – a code must be able to run stably under these conditions in order to be useful.

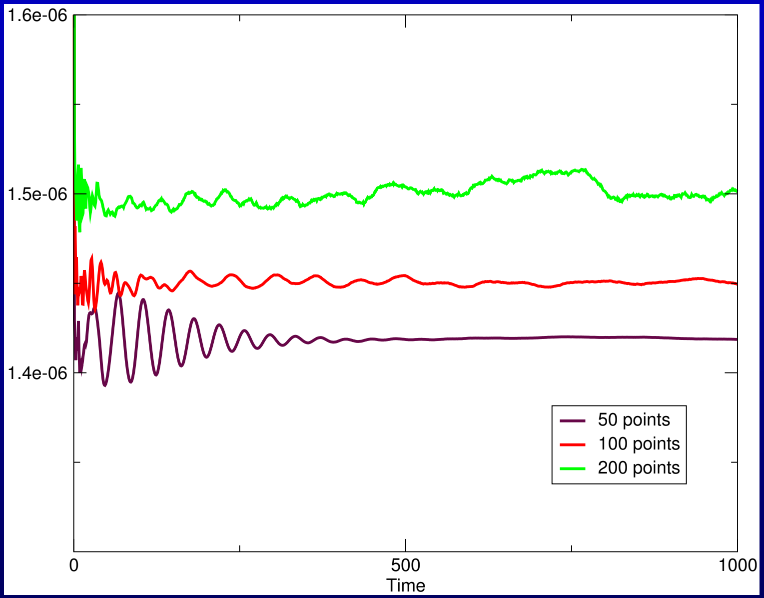
Typically the random noise is initially at the machine accuracy level, and takes some time to grow.

The robust stability test adds random noise noise to the initial data to accelerate the onset of instabilities.

Very useful for identifying weakly hyperbolic formulations: high resolutions will contain higher frequency perturbations, leading to resolution dependent results for weakly hyperbolic systems.

Robust stability: ADM vs. BSSN





ADM: Clear exponential growth. Increased resolution increases the growth rate.

Symptoms of weakly hyperbolicity.

BSSN: Only linear growth. Increased resolution does not change the growth.

Consistent with strong hyperbolicity.

Periodic gauge wave

A gauge wave is constructed by performing a transformation of the usual minkowski coordinates:

$$\hat{t} = t + \frac{Ad}{4\pi}\cos\left(\frac{2\pi(x-t)}{d}\right), \qquad \hat{x} = x - \frac{Ad}{4\pi}\cos\left(\frac{2\pi(x-t)}{d}\right)$$

This leads to the 4-metric:

$$ds^2 = -H d\hat{t}^2 + H d\hat{x}^2 + dy^2 + dz^2,$$

with $H = 1 - A\sin(2\pi(x-t)/d)$.

We specify these tests both in 1D (wave travelling along the x-axis), and 2D (wave propagating diagonally in the $x - y$ plane) via a coordinate transformation:

$$x = \frac{1}{\sqrt{2}}(x' - y'), \qquad y = \frac{1}{\sqrt{2}}(x' + y').$$

Periodic linear wave

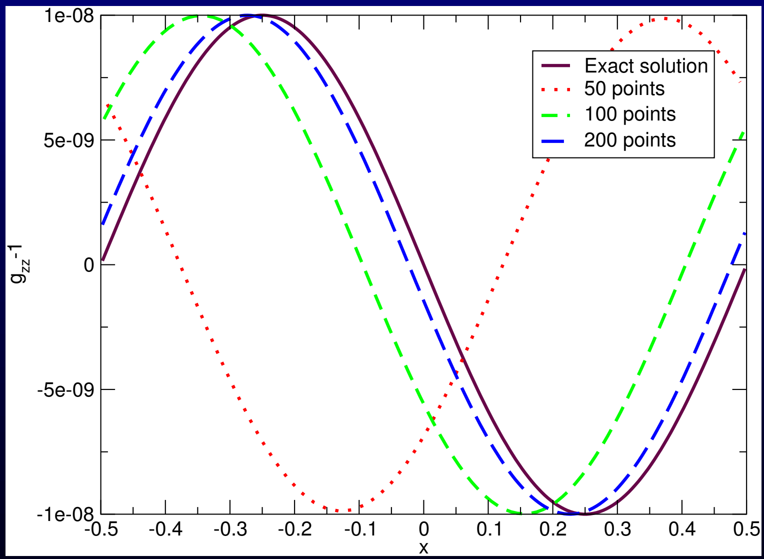
Consider a perturbation of flat space of the form:

$$ds^2 = -dt^2 + dx^2 + (1+b)dy^2 + (1-b)dz^2,$$

with

$$b = A \sin\left(\frac{2\pi(x-t)}{d}\right).$$

For small amplitude (eg. $A = 10^{-8}$) this solves the linearised Einstein equations and represents a plane wave travelling in the x-direction.



Polarised Gowdy spacetime

Solutions to Einstein's equations on a 3-torus, describing an expanding universe containing plane polarised waves.

$$ds^2 = t^{-1/2} e^{\lambda/2}(-dt^2 + dz^2) + t(e^P dx^2 + e^{-P} dy^2).$$

Einstein eqs can be solved to give $P(z,t)$, $\lambda(z,t)$ in terms of Bessel functions, eg.

$$P = J_0(2\pi t)\cos(2\pi z)$$

In the expanding direction, we can evolve with constant Δt, leading to exponential growth in g_{zz}.

In the collapsing direction, harmonic slicing ensures that the singularity is only reached at $t = -\infty$.

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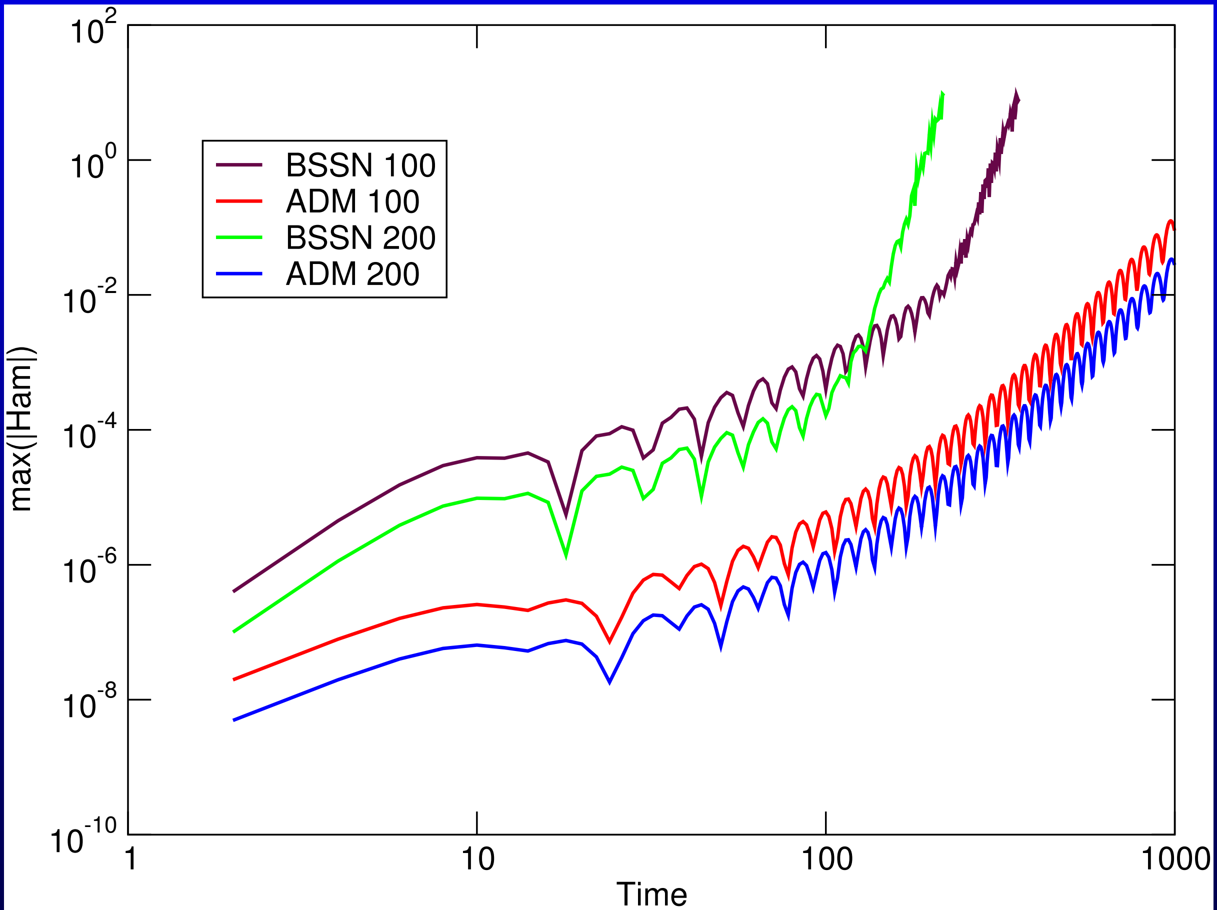

BSSN performs deceptively poorly in this test compared to ADM.

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It is interesting to study how different codes cope with these tests, however eventual crashes can be expected.

Whether this is of concern is debatable – spacetimes with a T^3 background are bound to be different from the physically interesting asymptotically flat cases.

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Does it make sense to loosen this restriction, to allow more flexibility for participating codes?

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(2) We wanted to isolate the problem to the evolution system: this restricts tests to models without boundaries.

Do we need to re-define the test systems as "formulations+BCs" rather than simply "formulations"?

- this recognises the fact that the two issues are inextricably linked

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In any case, a number of technical (and non-technical) issues need to be discussed first . . .