Comparing Spacetimes

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UNAM, Mexico, December 2003

Comparing Spacetimes – Why

We want to compare spacetimes that are given in different coordinate systems. Maybe the spacetimes are a slightly different, too.

- Compare runs with different gauge conditions, formulations, boundary conditions
- Ignore differences due to different discretisation errors, different amounts of constraint violations
- This might be difficult in full 4D. But we are interested in comparing initial as well.
- We need quantitative measurements.

Comparing Spacetimes – How

The following methods seem possible:

- Compare curvature invariants
- Evolve a set of invariant observers, and compare their trajectories in the coordinate systems of the spacetimes
- Locate invariant structures (e.g. apparent horizons, CMC surfaces) in the spacetimes and compare their coordinate shapes
- Determine explicitly a coordinate transformation between the spacetimes

In the Mexico spirit, I present ideas and working code.

Conventions

When comparing coordinate systems, there is a *reference system* and a *test system*.

Reference system: Given in advance; could be an analytic solution, or with a good gauge condition, or a high resolution run

Test system: The system under examination, i.e. the current run; might be a new gauge condition

Reference system quantities are given with an overbar, test system quantities without, as in: $\partial\bar{x}^\mu/\partial x^\nu$.

Not Done Here

- I don't consider curvature invariants here. They can be readily calculated in Cactus.
- I tried finding coordinate transformations via minimisation in $1+1$ spherical symmetry with an analytically given reference system. It works nicely with a good initial guess, but is a bit slow. It might be worth trying in $3+1$ dimensions.

Invariant Observers

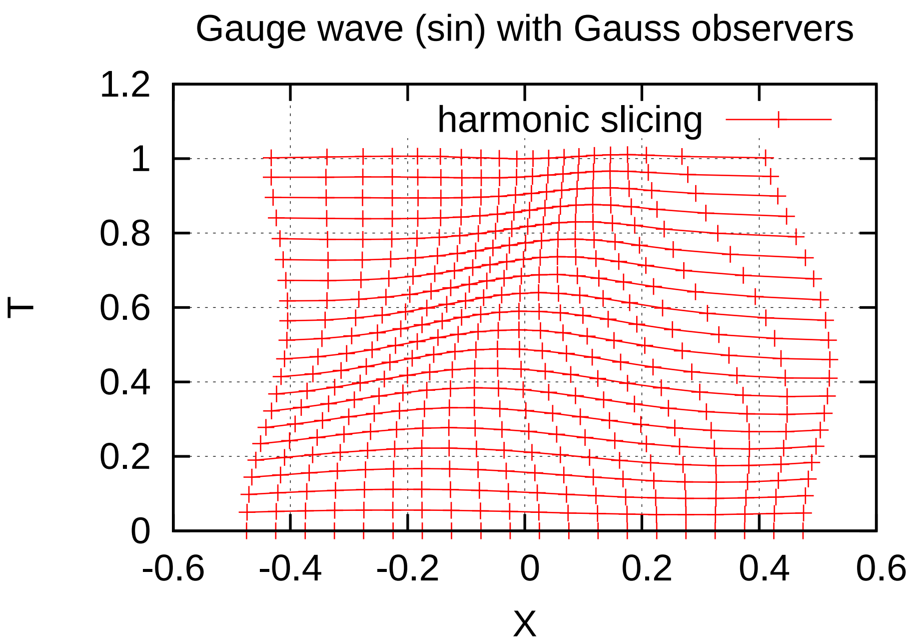
Idea (Hern, thesis, gr-qc/0004036): Evolve a set of reference observers with a certain lapse and shift together with the test system:

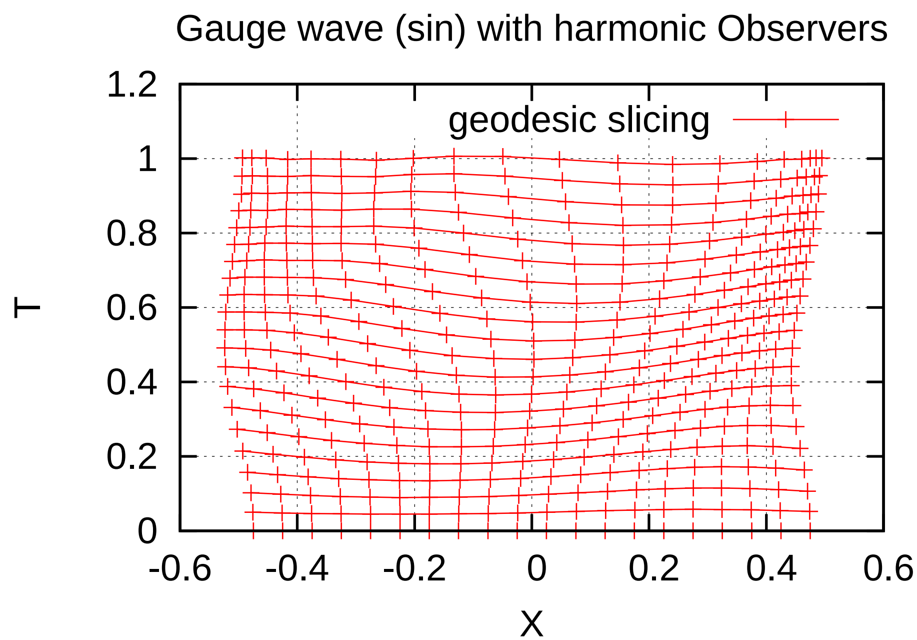
$$\bar{x}^\mu(t, x^i) \quad : \quad \bar{\alpha}, \bar{\beta}^i \longrightarrow \partial_t \bar{x}^\mu \tag{1}$$

(One evolves the reference coordinate system as function of the test coordinate system.)

- Initial data: It is in general not obvious how to set up initial data when the initial hypersurfaces do not coincide.
- The observer system is subject to different discretisation errors than a direct time evolution of the observer system. This leads to diverging errors.

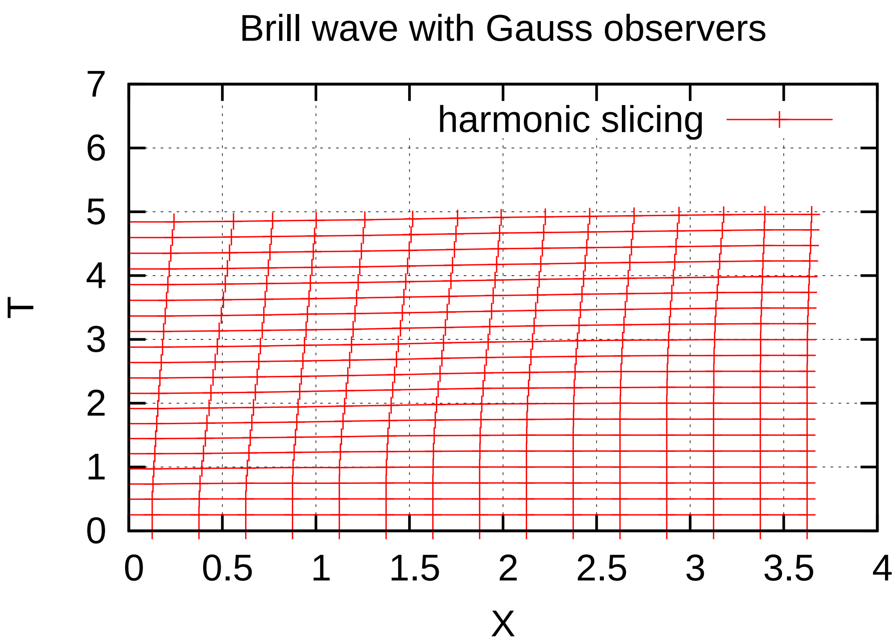
Observers, Example I: Gauge Wave

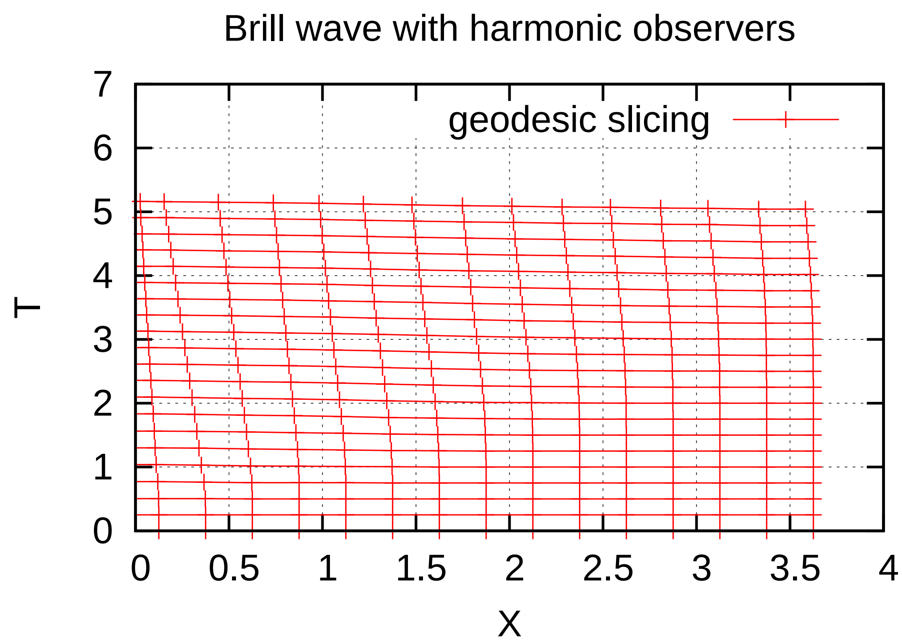




Gauge wave ($\sin$ type), $A = 0.25$, $dx = 1/20$

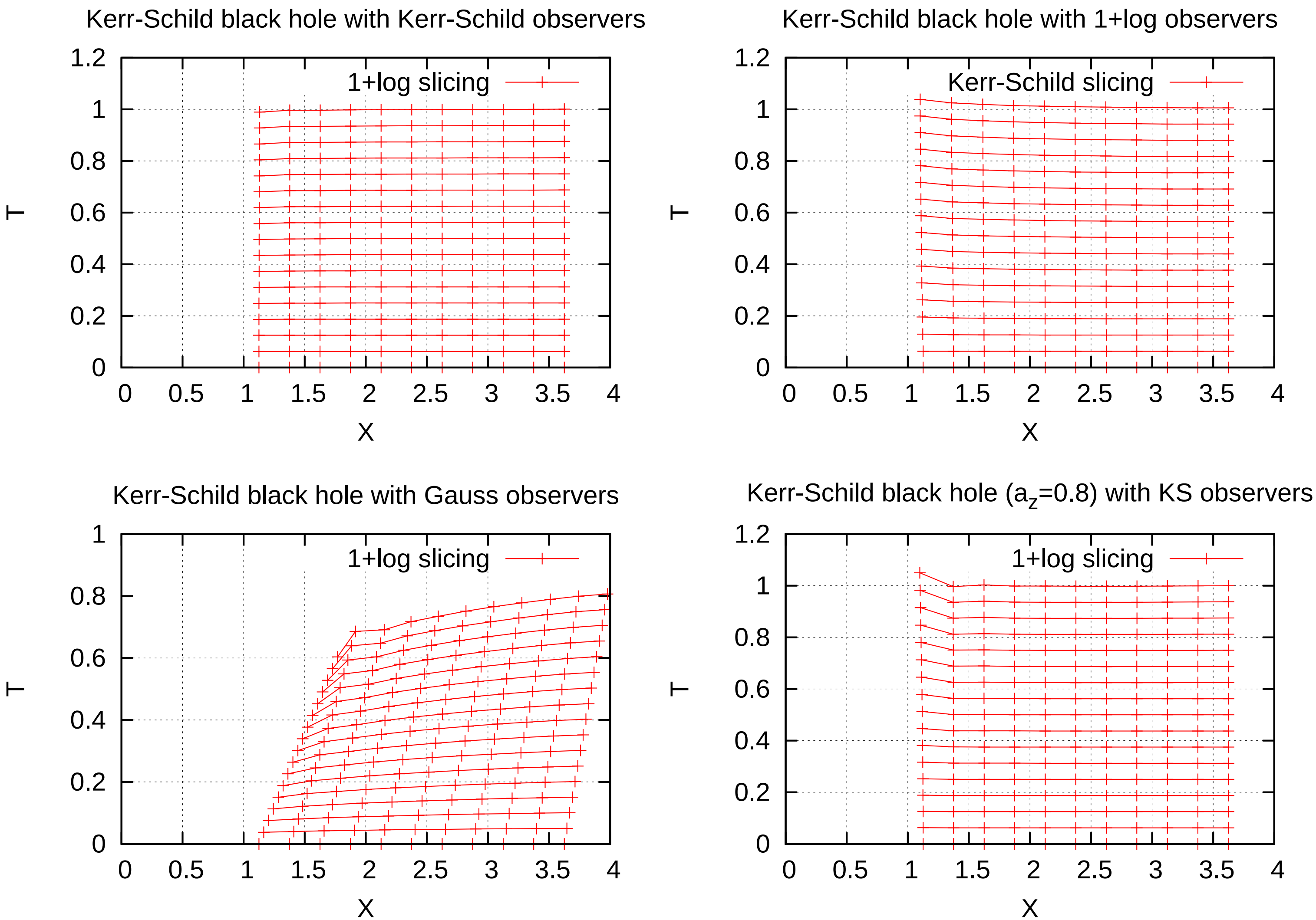
Observers, Example II: Brill Wave





Brill wave, $A = 1$, $dx = 1/4$

Observers, Example III: Black Hole



Observers, Conclusion

- Cheap to implement: a few more evolved quantities, nothing expensive
- Interesting results
- Discretisation errors tend to make observers diverge; hence not useful for long-term evolutions
- Cannot be used to compare initial data sets

Invariant Structures

This is difficult in 4D. I restrict myself to analysing 3D spacelike hypersurfaces. This assumes identical slicings, and is also useful for initial data.

I use *surfaces of constant expansion (CE)*, which are a generalisations of *surfaces of constant mean curvature (CMC)*. (CMC surfaces assume $K_{ij} = 0$.) CMC surfaces were used e.g. by Huisken and Yau to define the centre of mass of hypersurfaces. Apparent horizons are also CE surfaces. CE surfaces are slicing dependent. [Schnetter 2003]

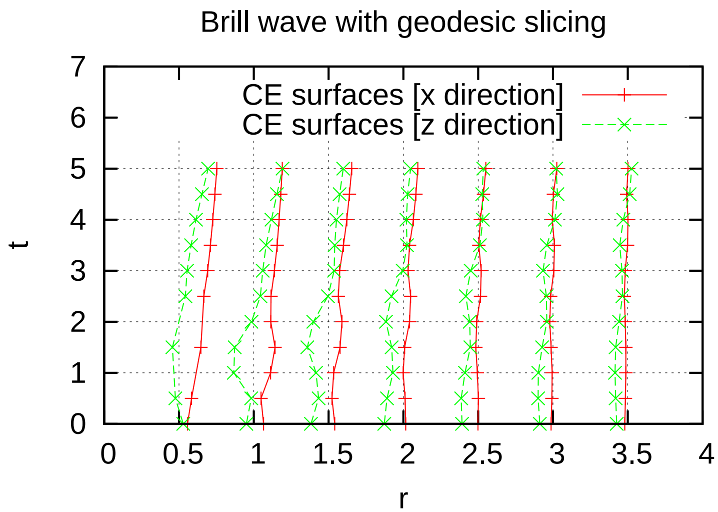
Invariant Structures II

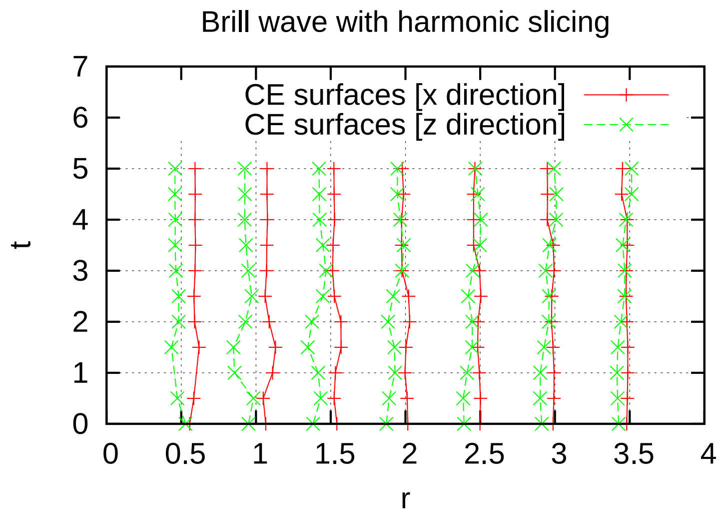
CE surfaces with given areas $A = 4\pi R^2$ define a radial coordinate R (outside of a region near the centre).

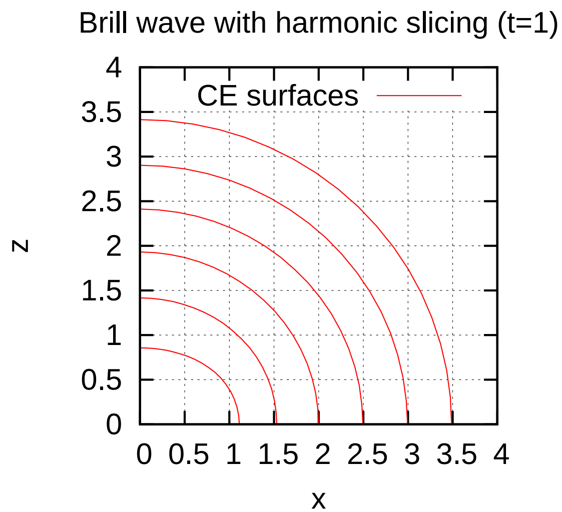
Left to do: Angular coordinates could e.g. be introduced by surface normals from $R \to \infty$. Another possibility is to look at Killing vector fields on the surfaces (if they exist). [Dreyer et al. 2003]

Coordinate distortion of CE surfaces measures the "coordinate quality". The physical distortion (proper lengths of geodesics) is independent of the spatial gauge.

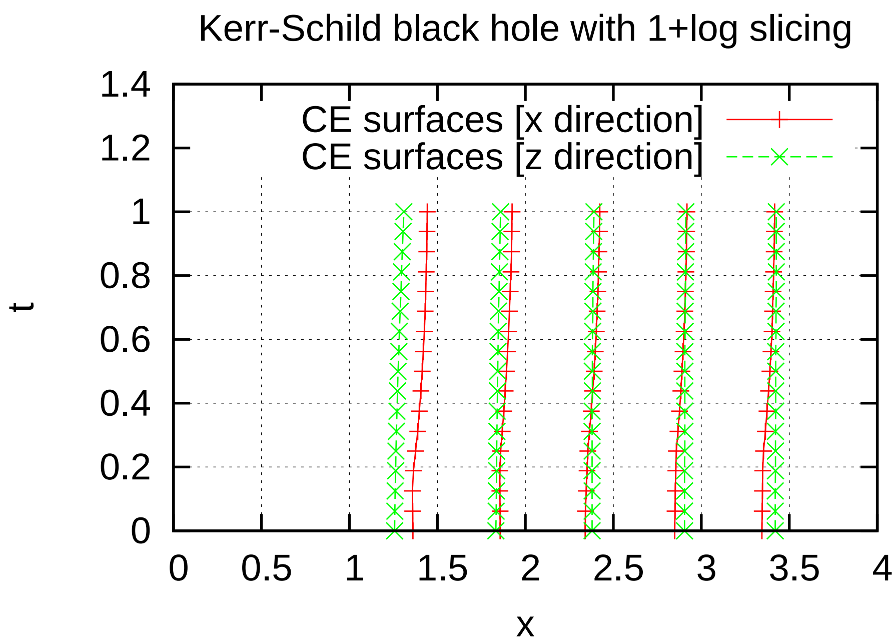
Structures, Example I: Brill Wave

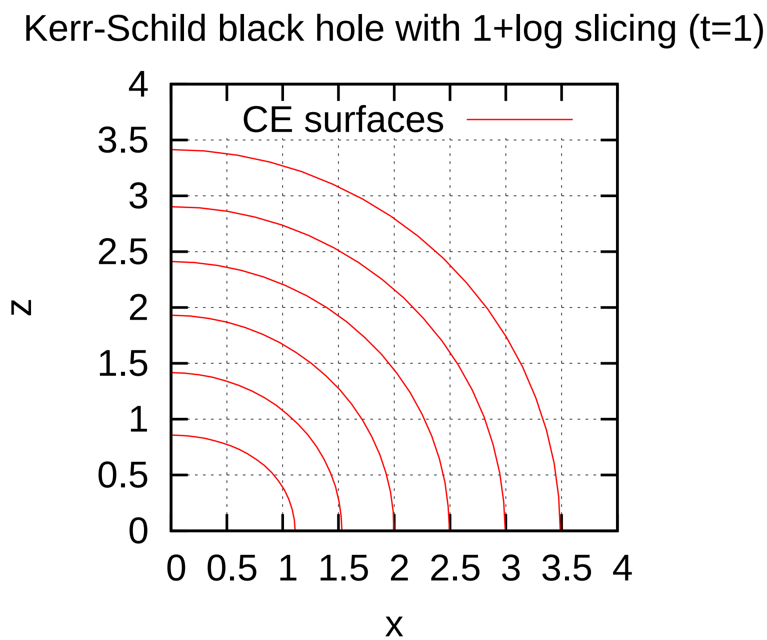






Structures, Example II: Black Hole





Structures, Conclusion

- Each hypersurface is analysed independent of the others; no discretisation errors accumulate
- Finding a set of CE surfaces is a bit expensive (similar to apparent horizons)
- Can be run only every n time steps, or as postprocessing step
- Interesting results
- Especially useful for initial data

Towards Mexico II

Will there be a second set of standard testbeds?

If so, it could suggest to compare observers or CE surfaces.