Measuring Waves on a Numerical Grid

Frank Herrmann

Overview

- Introduction
- Wave Extraction methods in Cactus
  - Near Zone
  - Far Zone
- Results
  - Binary Black Hole Collisions
    - head on
    - ISCO

Introduction

- Wave Extraction can be used as a physics code test
  - Compare physics predictions from different codes
  - ... and even to reality
- more of a longterm goal

Gravitational Radiation

- gravitational radiation defined only in radiation zones
- where geometry is defined by 2 length scales
  1. average radius of curvature
  2. much shorter ripples
- Natural way to think about waves is as perturbation on background
- Potential trouble with this picture
  - many backgrounds might exist
  - how do you choose

Methods used for Wave Analysis

- Wave analysis tools implemented in Cactus
  - Near Zone
    - Horizon Oscillations
  - Far Zone
    1. Regge-Wheeler-Zerilli-Moncrief (Schwarzschild)
    2. Newman-Penrose Ψ_4
       - numerically defined radial tetrad
       - Kinnersley tetrad (choose a Kerr background)
    3. Other measures
       - Radiation Scalar (Beetle and Burko 2002)
- ... not all work equally well in all situations ...

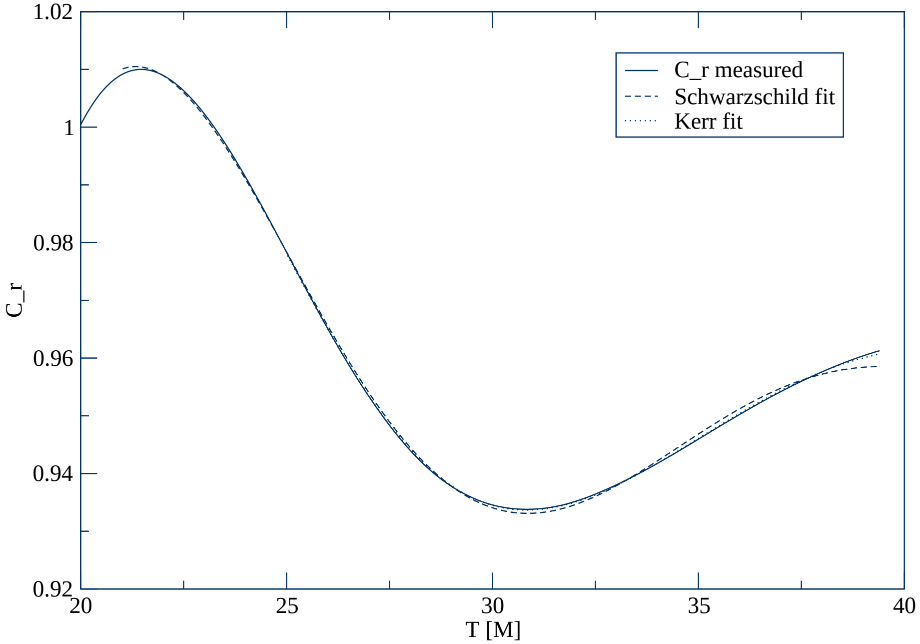
Horizon Oscillations

- Measure polar and equatorial circumferences of apparent horizon C_p, C_e
- Use $C_r = C_p/C_e$ as a measure for horizon dynamics
- Shows quasi-normal mode ringing
  - Gravitational radiation created at peak of Zerilli potential
  - Waves travel in and out
  - in-travelling waves interact with horizon

Horizon Oscillations

- Coordinate dependent measure
- frequency should be coordinate dependent
- works surprisingly well in ring-down phase

Horizon Oscillations



Apparent Horizon Oscillations of final merged black hole

ISCO: $C_r =$ Polar$/$Equatorial Circumference

Perturbative Approaches

- Regge-Wheeler Zerilli Moncrief
  - Metric based perturbation theory around Schwarzschild background
- Teukolsky
  - Newman-Penrose based perturbation theory around Kerr background

Zerilli Extraction

- metric perturbative approach $g_{\mu\nu} = g^S_{\mu\nu} + \epsilon h_{\mu\nu}$
- decompose metric into spherical harmonics (even parity)

$$g^S_{\mu\nu} = \begin{pmatrix} -N^2 & 0 & 0 & 0 \\ 0 & A^2 & 0 & 0 \\ 0 & 0 & R^2 & 0 \\ 0 & 0 & 0 & R^2 \sin^2\theta \end{pmatrix}$$

$$h_{\mu\nu} = \begin{pmatrix} N^2 H_0 Y_{l0} & H_1 Y_{l0} & h_0 Y_{l0,\theta} & 0 \\ H_1 Y_{l0} & A^2 H_2 Y_{l0} & h_1 Y_{l0,\theta} & 0 \\ h_0 Y_{l0,\theta} & h_1 Y_{l0,\theta} & R^2 (K + G \frac{\partial^2}{\partial\theta^2}) Y_{l0} & 0 \\ 0 & 0 & 0 & R^2 (K \sin^2\theta + G \sin\theta\cos\theta \frac{\partial}{\partial\theta} Y_{l0}) \end{pmatrix}$$

- Construct 1st order gauge invariant quantities [Moncrief 1974] (even parity)

$$
\begin{aligned}
k_1 &= K + SrG_{,r} - 2\frac{S}{r}h_1 \\
k_2 &= \frac{H_2}{2S} - \frac{1}{2\sqrt{S}}\frac{\partial}{\partial r}\left(rS^{-1/2}K\right) \\
S &= 1 - 2M/r \\
\Lambda &= l(l+1) - 2 + \frac{6M}{r} \\
\psi_Z &= \sqrt{\frac{2(l-1)(l+2)}{l(l+1)}}\frac{4rS^2k_2 + l(l+1)rk_1}{\Lambda}
\end{aligned}
$$

  – assume spherical background, i.e. not Kerr
  – extension to Kerr might be possible, but very hard

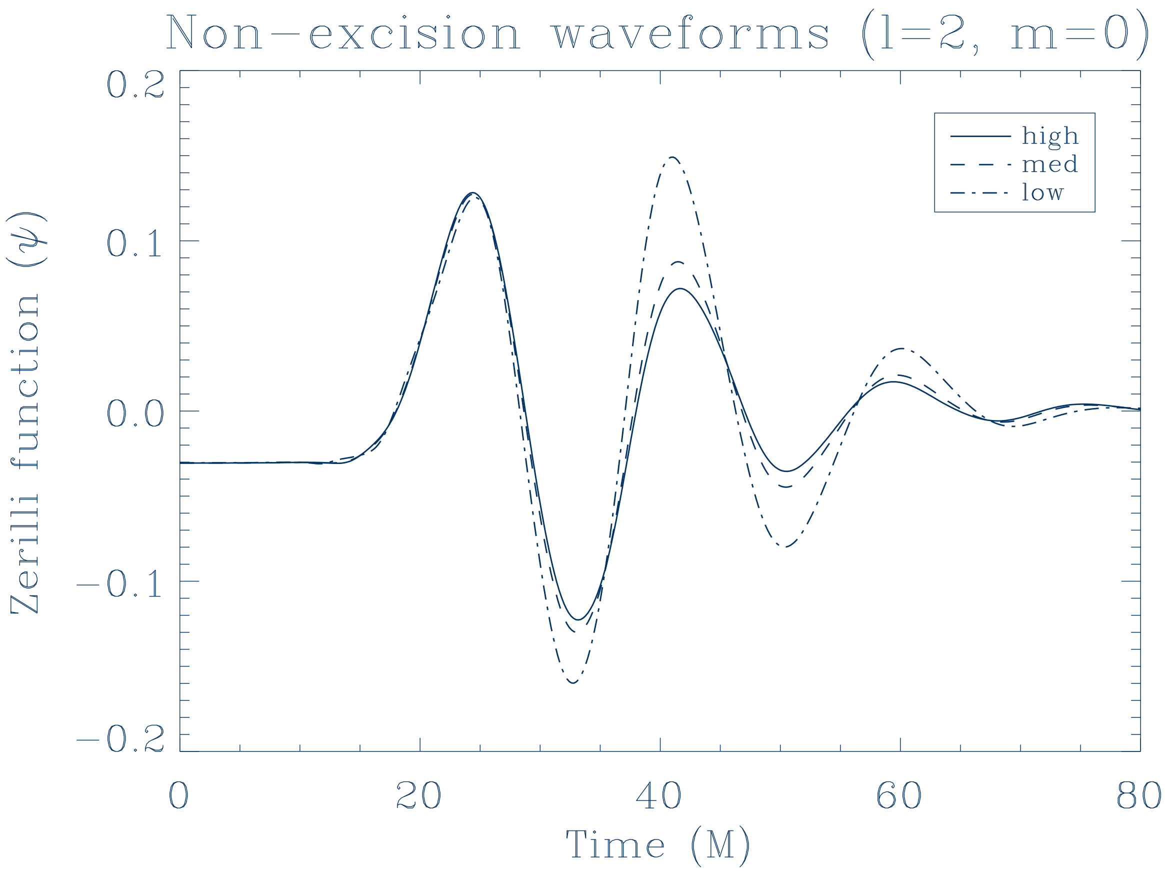
- Advantages

  – very accurate for Schwarzschild background

- works surprisingly well even for rotating distorted black holes

- Disadvantages
  - only 1st order gauge invariant (strong gauge effects)
  - needs spherically symmetric background
- Perturbative Matching to Zerilli Equation possible
- Placement of detectors (extraction spheres) is important
  - far-out little influence from background, but then the outer boundary makes trouble
  - this can also be used to monitor dirt in the simulation as it crosses the detectors

Zerilli Extraction



Head-on collision (Peter Diener)

Newman-Penrose Ψ_4

- Choose tetrad frame (null vectors) $\mathbf{l}, \mathbf{n}, \mathbf{m}, \overline{\mathbf{m}}$ with

$$l^a n_a = -m^a \overline{m}_a = -1$$

- Calculate Weyl scalars ($\Psi_0 \ldots \Psi_4$)

$\Psi_0 = C_{abcd}\, l^a\, m^b\, l^c\, m^d$	(transverse radiation along n^a)
$\Psi_1 = C_{abcd}\, l^a\, m^b\, l^c\, n^d$	(longitudinal radiation along n^a)
$\Psi_2 = C_{abcd}\, l^a\, m^b\, \overline{m}^c\, n^d$	(Coulombic field)
$\Psi_3 = C_{abcd}\, l^a\, n^b\, \overline{m}^c\, n^d$	(longitudinal radiation along l^a)
$\Psi_4 = C_{abcd}\, \overline{m}^a n^b\, \overline{m}^c\, n^d \overline{m}^d$	(transverse radiation along l^a)

- Coordinate invariant, but tetrad dependent
- $\Psi_4 \sim 1/r$ and $\Psi_0 \sim 1/r^5$ (tetrad dependent peeling property)
- $\Psi_4 \rightarrow R_{t\theta t\theta} + iR_{t\theta t\phi}$

Teukolsky Extraction

- as done by Lazarus Group
- Kerr background
- Boyer-Lindquist coordinates
- Kinnersley (background specific) tetrad used
- Newman-Penrose quantities can then be evolved with Teukolsky equation
- fairly robust under choice of background (a,M parameters)

Non-Perturbative

- Newman-Penrose scalars
- other quantities which provide wave measure in radiation zone

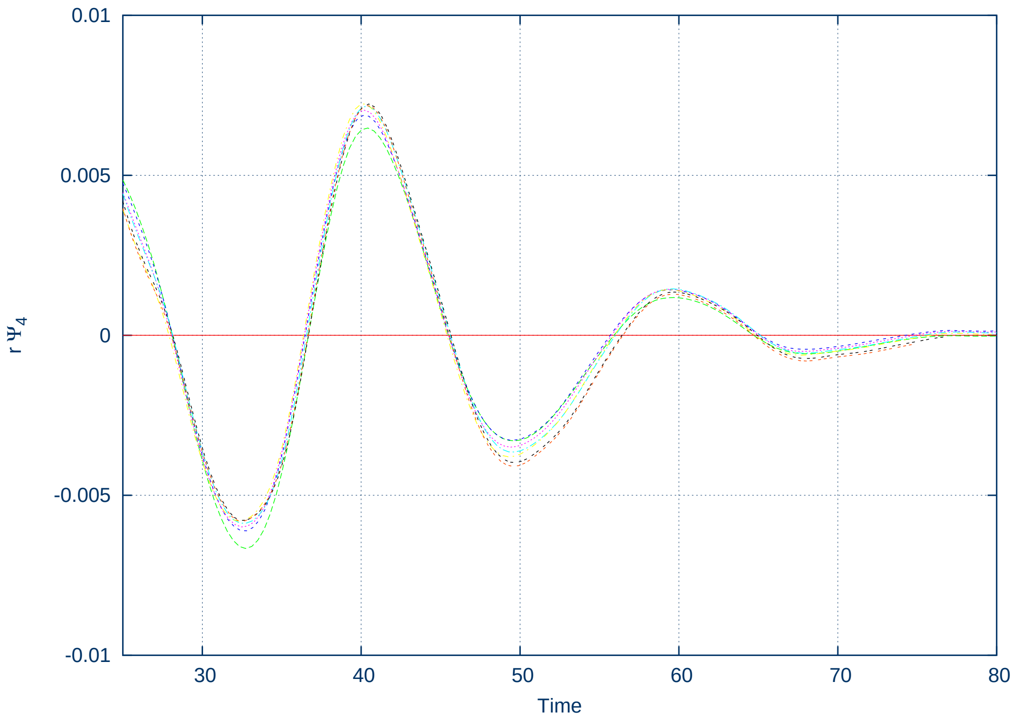
Newman-Penrose Ψ_4

- tetrad frame (null vectors) $\mathbf{l}, \mathbf{n}, \mathbf{m}, \overline{\mathbf{m}}$
- ingoing $n^a = 1/\sqrt{2}(u^a - r^a)$
- outgoing $l^a = 1/\sqrt{2}(u^a + r^a)$
- spacelike 2-sphere spawned by $m^a = 1/\sqrt{2}(\theta^a + i\phi^a)$, $\overline{m}^a$
- $\Psi_4 \sim 1/r$ and $\Psi_0 \sim 1/r^5$ (tetrad dependent peeling property)
- $\Psi_4 \rightarrow R_{t\theta t\theta} + iR_{t\theta t\phi}$

Newman-Penrose Ψ_4

- Note that non-vanishing Ψ_0 is important to distinguish from pure gauge wave
- check for $1/r$ fall-off behaviour in Ψ_4
- special tetrad frame (Example: $\Psi_1 = \Psi_3 = 0$)

Head-On $\Re(r\Psi_4)$



(l=2, m=0) QNM 16.8 M shifted so that peak matches

Further possibilities

- I or J invariants and quantites constructed from those
- radiation scalar $\xi = \Psi_0^T \Psi_4^T$ (Beetle & Burko 2002)
- Cauchy-Characteristic extraction

Issues and Outlook

- Comparison of different scenarios for different wave indicators necessary
- Likely best indicator is model dependent
- Zerilli works very well for non-rotation cases