

# Measuring Waves on a Numerical Grid

Frank Herrmann

Frank Herrmann/AEI 2003



# Overview

- Introduction
- Wave Extraction methods in Cactus
  - Near Zone
  - Far Zone
- Results
  - Binary Black Hole Collisions
    - \* head on
    - \* ISCO



# Introduction

- Wave Extraction can be used as a physics code test
  - Compare physics predictions from different codes
  - ... and even to reality
- more of a longterm goal



# Gravitational Radiation

- gravitational radiation defined only in radiation zones
- where geometry is defined by 2 length scales
  - 1 average radius of curvature
  - 2 much shorter ripples
- Natural way to think about waves is as perturbation on background
- Potential trouble with this picture
  - many backgrounds might exist
  - how do you choose



# Methods used for Wave Analysis

- Wave analysis tools implemented in Cactus
  - Near Zone
    - \* Horizon Oscillations
  - Far Zone
    1. Regge-Wheeler-Zerilli-Moncrief (Schwarzschild)
    2. Newman-Penrose  $\Psi_4$ 
      - numerically defined radial tetrad
      - Kinnersley tetrad (choose a Kerr background)
    3. Other measures
      - Radiation Scalar (Beetle and Burko 2002)
- ... not all work equally well in all situations ...



# Horizon Oscillations

- Measure polar and equatorial circumferences of apparent horizon  $C_p, C_e$
- Use  $C_r = C_p/C_e$  as a measure for horizon dynamics
- Shows quasi-normal mode ringing
  - Gravitational radiation created at peak of Zerilli potential
  - Waves travel in and out
  - in-travelling waves interact with horizon

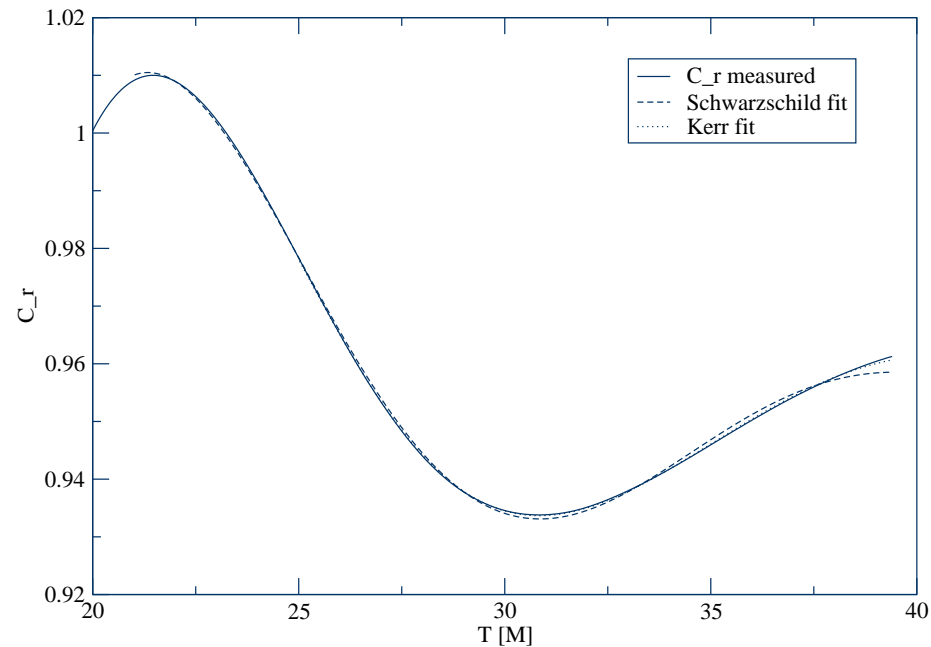


# Horizon Oscillations

- Coordinate dependent measure
- frequency should be coordinate dependent
- works surprisingly well in ring-down phase



# Horizon Oscillations



Apparent Horizon Oscillations of final merged black hole  
ISCO:  $C_r = \text{Polar/Equatorial Circumference}$



# Perturbative Approaches

- Regge-Wheeler Zerilli Moncrief
  - Metric based perturbation theory around Schwarzschild background
- Teukolsky
  - Newman-Penrose based perturbation theory around Kerr background



# Zerilli Extraction

- metric perturbative approach  $g_{\mu\nu} = g_{\mu\nu}^S + \epsilon h_{\mu\nu}$
- decompose metric into spherical harmonics (even parity)

$$g_{\mu\nu}^S = \begin{pmatrix} -N^2 & 0 & 0 & 0 \\ 0 & A^2 & 0 & 0 \\ 0 & 0 & R^2 & 0 \\ 0 & 0 & 0 & R^2 \sin^2 \theta \end{pmatrix}$$

$$h_{\mu\nu} = \begin{pmatrix} N^2 H_0 Y_{l0} & H_1 Y_{l0} & h_0 Y_{l0,\theta} & 0 \\ H_1 Y_{l0} & A^2 H_2 Y_{l0} & h_1 Y_{l0,\theta} & 0 \\ h_0 Y_{l0,\theta} & h_1 Y_{l0,\theta} & R^2 (K + G \frac{\partial^2}{\partial \theta^2}) Y_{l0} & 0 \\ 0 & 0 & 0 & R^2 (K \sin^2 \theta + G \sin \theta \cos \theta \frac{\partial}{\partial \theta} Y_{l0}) \end{pmatrix}$$



- Construct 1st order gauge invariant quantities [Moncrief 1974] (even parity)

$$k_1 = K + SrG_{,r} - 2\frac{S}{r}h_1$$

$$k_2 = \frac{H_2}{2S} - \frac{1}{2\sqrt{S}}\frac{\partial}{\partial r}\left(rS^{-1/2}K\right)$$

$$S = 1 - 2M/r$$

$$\Lambda = l(l+1) - 2 + \frac{6M}{r}$$

$$\psi_Z = \sqrt{\frac{2(l-1)(l+2)}{l(l+1)}}\frac{4rS^2k_2 + l(l+1)rk_1}{\Lambda}$$

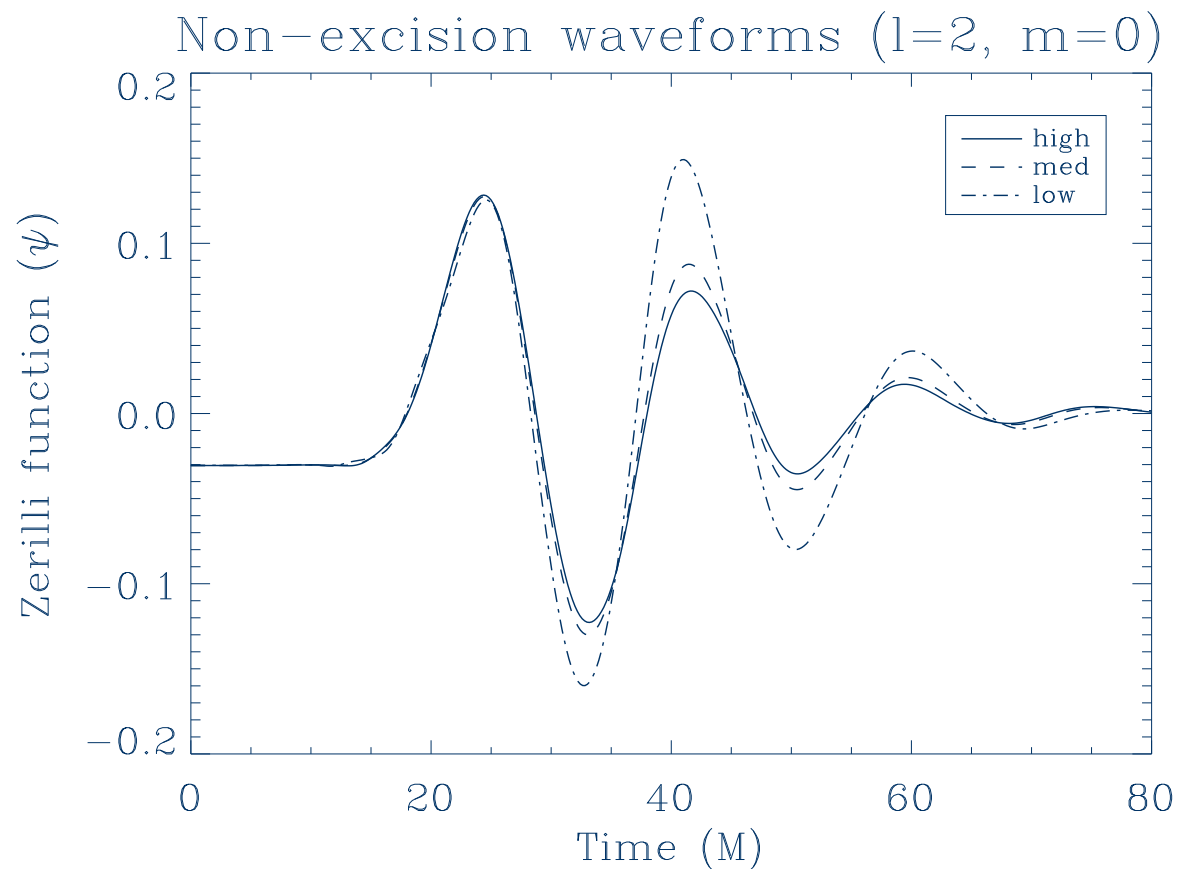
- assume spherical background, i.e. not Kerr
- extension to Kerr might be possible, but very hard
- Advantages
  - very accurate for Schwarzschild background



- works surprisingly well even for rotating distorted black holes
- Disadvantages
  - only 1st order gauge invariant (strong gauge effects)
  - needs spherically symmetric background
- Perturbative Matching to Zerilli Equation possible
- Placement of detectors (extraction spheres) is important
  - far-out little influence from background, but then the outer boundary makes trouble
  - this can also be used to monitor dirt in the simulation as it crosses the detectors



# Zerilli Extraction



Head-on collision (Peter Diener)



# Newman-Penrose $\Psi_4$

- Choose tetrad frame (null vectors)  $l, n, m, \bar{m}$  with

$$l^a n_a = -m^a \bar{m}_a = -1$$

- Calculate Weyl scalars ( $\Psi_0 \dots \Psi_4$ )

$$\Psi_0 = C_{abcd} l^a m^b l^c m^d \quad (\text{transverse radiation along } n^a)$$

$$\Psi_1 = C_{abcd} l^a m^b l^c n^d \quad (\text{longitudinal radiation along } n^a)$$

$$\Psi_2 = C_{abcd} l^a m^b \bar{m}^c n^d \quad (\text{Coulombic field})$$

$$\Psi_3 = C_{abcd} l^a n^b \bar{m}^c n^d \quad (\text{longitudinal radiation along } l^a)$$

$$\Psi_4 = C_{abcd} \bar{m}^a n^b \bar{m}^c n^d \bar{m}^d \quad (\text{transverse radiation along } l^a)$$



- Coordinate invariant, but tetrad dependent
- $\Psi_4 \sim 1/r$  and  $\Psi_0 \sim 1/r^5$  (tetrad dependent peeling property)
- $\Psi_4 \rightarrow R_{t\theta t\theta} + iR_{t\theta t\phi}$



# Teukolsky Extraction

- as done by Lazarus Group
- Kerr background
- Boyer-Lindquist coordinates
- Kinnersley (background specific) tetrad used
- Newman-Penrose quantities can then be evolved with Teukolsky equation
- fairly robust under choice of background ( $a, M$  parameters)



# Non-Perturbative

- Newman-Penrose scalars
- other quantities which provide wave measure in radiation zone



# Newman-Penrose $\Psi_4$

- tetrad frame (null vectors)  $l, n, m, \bar{m}$
- ingoing  $n^a = 1/\sqrt{2}(u^a - r^a)$
- outgoing  $l^a = 1/\sqrt{2}(u^a + r^a)$
- spacelike 2-sphere spawned by  $m^a = 1/\sqrt{2}(\theta^a + i\phi^a)$ ,  
 $\bar{m}^a$
- $\Psi_4 \sim 1/r$  and  $\Psi_0 \sim 1/r^5$  (tetrad dependent peeling property)
- $\Psi_4 \rightarrow R_{t\theta t\theta} + iR_{t\theta t\phi}$

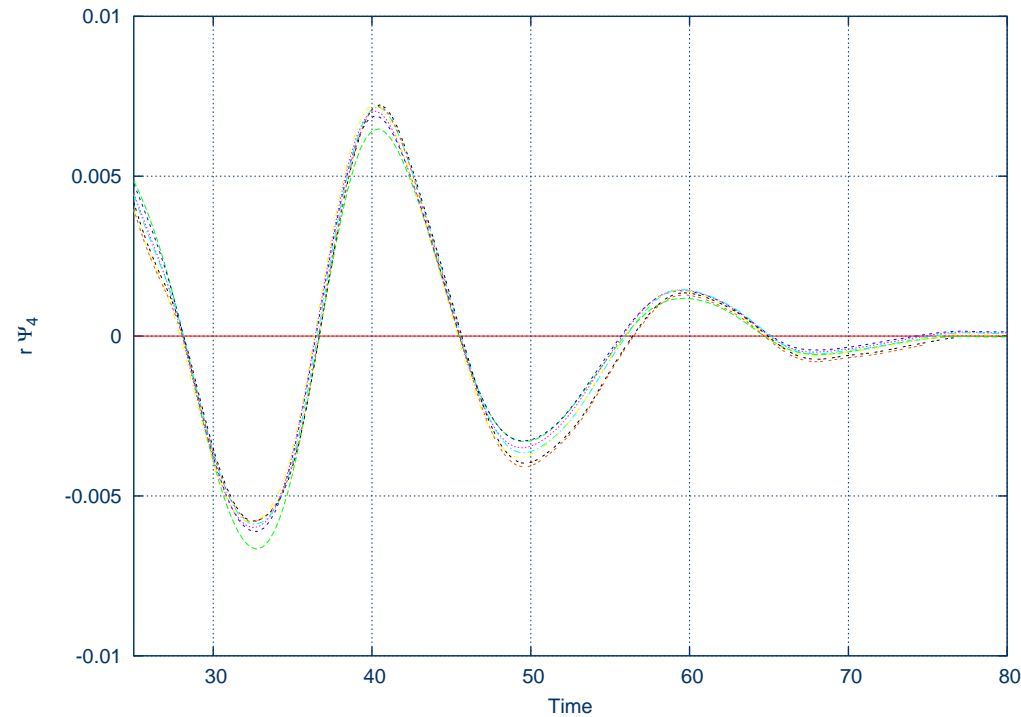


# Newman-Penrose $\Psi_4$

- Note that non-vanishing  $\Psi_0$  is important to distinguish from pure gauge wave
- check for  $1/r$  fall-off behaviour in  $\Psi_4$
- special tetrad frame (Example:  $\Psi_1 = \Psi_3 = 0$ )



# Head-On $\Re(r\Psi_4)$



( $l=2, m=0$ ) QNM 16.8 M shifted so that peak matches



# Further possibilities

- $I$  or  $J$  invariants and quantities constructed from those
- radiation scalar  $\xi = \Psi_0^T \Psi_4^T$  (Beetle & Burko 2002)
- Cauchy-Characteristic extraction



# Issues and Outlook

- Comparison of different scenarios for different wave indicators necessary
- Likely best indicator is model dependent
- Zerilli works very well for non-rotation cases